THE GEOMETRY of THE PANTHEON'S VAULT

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1 Geometry of the Vault

From the point of view of perspective, the only noteworthy item of the interior of the vault of the Pantheon is the visual effect produced by the geometric composition of the coffers. In order to draw the perspective outline of the interior of the Pantheon, it was essential to first understand how the vault was built, since the presence of the coffers did not arise from a decorative idea but rather from the need to reduce the weight of the vault, while maintaining a cross-section that was sufficiently robust to support its own weight. If we assume the hypothesis that the vault was built using an arched framework supported by the great cornice and set out radially, then the constructive outline of the coffers must have been controlled by means of molds built on the framework itself. In addition, it is possible that such construction was carried out at the floor level using a template that describes at least half of the hemisphere.

Palladio interpreted the inside slopes of the coffered ceiling of the vault cross-section as a radial outline at the level of the great cornice (see **Figure 1**). In surveys subsequent to those of Palladio, as well as in recent photographic studies, it can be seen that the inferior soffit slopes of the mitered joints that form the coffered ceiling seem to be directed towards the floor, that is, the view is flush with respect to the inferior soffit planes, while the superior soffit slopes seem to follow the radial outline at the level of the cornice. In order to determine the geometric outline of the coffers, it is essential to obtain the exact measurement in-situ. Such geometric evidence was not available for this study. However, our objective was not to obtain a perspective that faithfully represented the real construction, but rather to demonstrate how to approach a complex problem of perspective outline using the generative methods of distance vanishing point and diagonal vanishing point. In view of this, we considered it was sufficient to base our study on the information obtained from the drawing of W. L. MacDonald [1], as well as on photographs taken and observations made in-situ by the author (02.1994).

There are two explanations for the emphasized construction of the inferior soffit slopes of the coffered ceiling of the vault. The first explanation suggests that this particularity might be due to the constructive procedure itself, while the second explanation suggests that there may have been a deliberate intention of formal appearance. In the first case, it is possible that Agippa emphasized

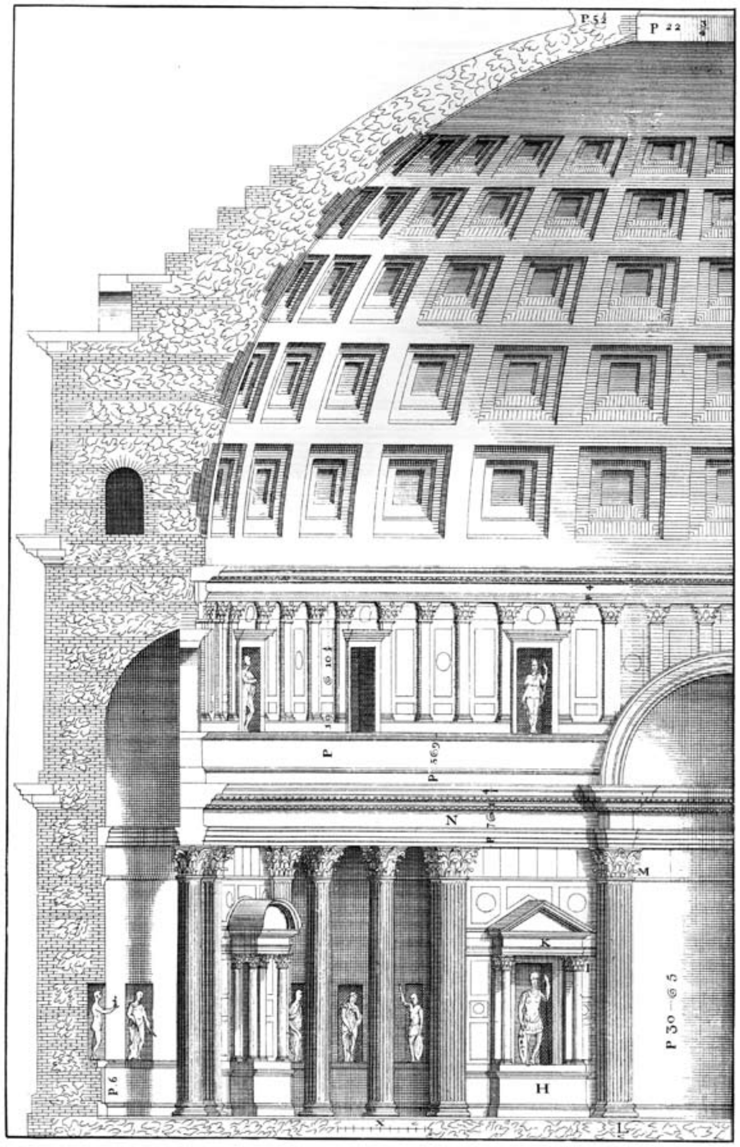

Figure 1. Andrea Palladio, Pantheon's Section from *I Quattro Libri dell'Architettura*, LVII.

the inferior soffit slope vertically, to ensure the stability of the casting of the concrete, assuming that the vault was structured and poured from the base up until it was closed at the oculus. In the second case, if the form that was adopted did not follow from a constructive consideration, it is possible that the intention of the aspect of the coffered ceiling was to avoid that the soffits remain hidden from the observer, thus avoiding an unaesthetic appearance and the accumulation of dust. In this respect, and based on a detailed photograph of the coffered ceiling (see Photograph 1), Henri Stierlin [2] commented: "Note the subtlety of the displacement toward the top of the strengthening elements in order to correct the optical distortion due to the observer's position on the ground and not at the level of the hemisphere".

Photograph 1, Henri Stierlin, see reference [2].

An analysis of Stierlin's photograph (see **photo 1**), by means of **Figure 2**, leads to infer that all the coffered ceilings have a trapezoidal outline that can be made to fit in a square if the length of the longer base is averaged with the length of the shorter base. Since we do not know if the photograph was taken from a position perpendicular to the plane of the coffered ceiling, it is impossible to be certain that the analytic outline is correct. However, the photograph is demonstrative with respect to the top shift of the center of the outline from (*c*) to (*c*') for the inferior soffit slopes. In the same figure, it can be observed that the outline of the superior soffit slopes of the coffered ceiling tends to follow the reference of the upper diagonals of the trapezium.

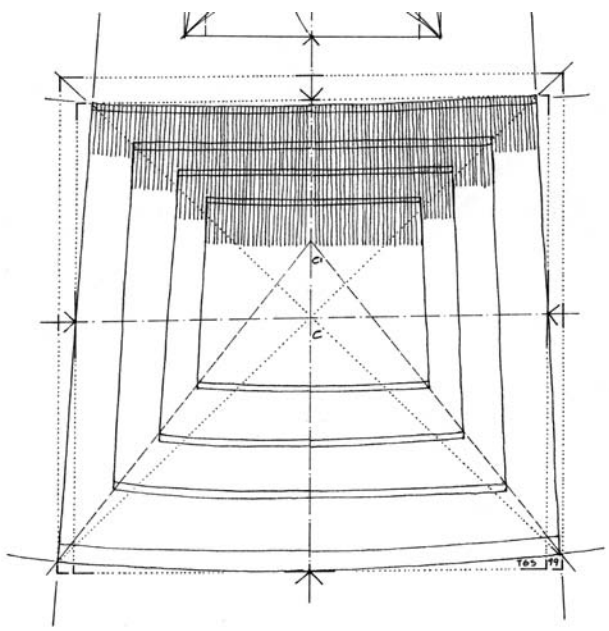

Figure 2. Geometrical analisys of the coffered ceilings by TGS.

The reason for assuming that the coffered ceilings arise from a square whose base is governed by the diminishing outline of the 28 meridians, is that the coffered ceilings do not correspond to a readily identifiable constant or progressive radial outline, whether referred to the center of the sphere or to its projection on the floor. Therefore, our hypothesis for the outline of five courses of the coffered ceilings assumes that their construction followed a physical deduction on the meridian lines, based on some simple geometric rule for the outline of the framework of the coffered ceilings. In Fig. 6, if the profile of the coffer of the first course is taken as a reference and then shifted successively to the others, it can be observed that the inferior and superior soffit slopes tend to be the same.

This suggests that some type of master mold was used to build the framework and that it was simply adjusted to the dimensions of each meridian course. Assuming that a measurement in-situ were to show that the slopes are the same, the reason for emphasizing the inferior soffit slopes would still remain unexplained. Although we have suggested two possible explanations from the constructive and formal points of view, neither is sufficiently demonstrative. So, for the time being, we will let the Pantheon keep some of the secrets of its construction.

2 Perspective of the Vault

Several treatise writers, painters and geometrists have attempted the outline of the interior views of the Pantheon. Giovanni Paolo (c. 1750), Pannini, Gian and Francesco Piranesi (XVIII century) made drawings and engravings of such a high quality that they are considered to be historical documents. Among the writers, Piero della Francesca studied a problem similar to that of the construction of the perspective of the Pantheon [3]. Although the dimensions of his problem were smaller, it is interesting to analyse his method of outline. However, after studying the outlines of these works, we concluded that none of them offer an exact geometric description. This may be because their designers did not follow a method that would allow them to directly control the perspective outline. The traditional methods of perspective become very intricate and inexact when applied to deducing subtleties such as the sweep of the vault soffits, in other words, when generating the perspective geometry.

In view of the above, we became interested in the perspective outline of the vault, with the purpose of testing a method that would simplify its execution. However, before attempting any outline based on the hypothesis of formal appearance or constructive procedure, we needed to establish the geometric construction of the coffered ceiling in a consistent manner. To this end, we found it very useful to study not only the historical background of the Pantheon but also the Roman architecture of the time, when vault-building technology reached its climax.

Agrippa or Hadrian solved by some means the problem for the construction of the framework of the coffered ceiling, to achieve their uniformity, which, as mentioned above, suggests the use of molds. However, such molds must have been based on an established outline or pattern. When faced with a similar problem in my practice as an architect, I have always sought that the outline of something that is to be built repeatedly be simple to execute. For example, if we had to determine the outline of the five master frameworks that form a meridian course, the first thought would be to use a sole principle to generate the entire woodwork. The principle would be simple, easy to apply in situ, and deduced as a function of the decreasing value of the meridian course. This reasoning can be explained in the following manner by means of **Figure 3**.

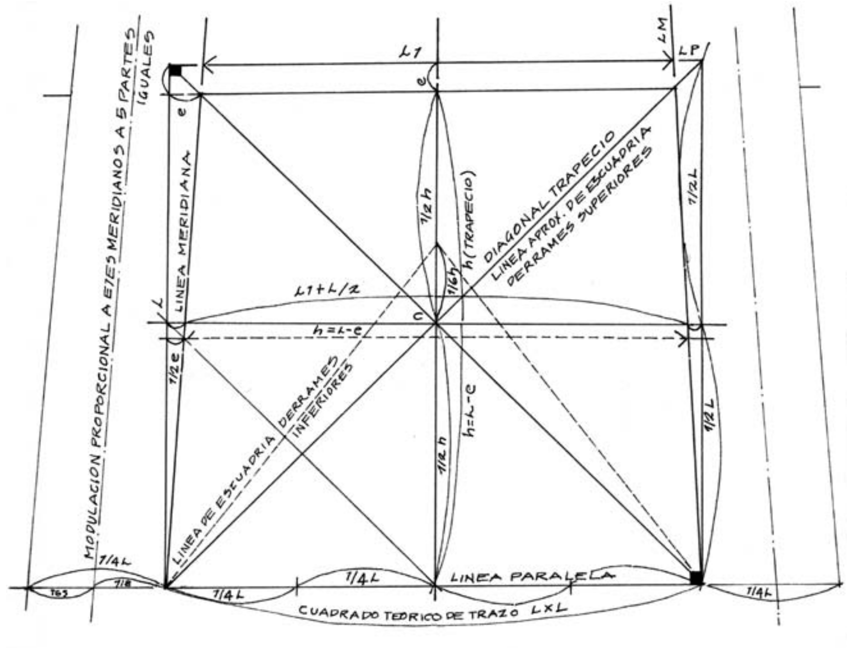

Figure 3. The outline principle to generate the woodwork, hypothesis by TGS.

2.1 Assuming that the intention of the outline [4] is to design the coffered ceiling with a proportional appearance close to that of a square, whether slightly larger or smaller than the trapezium to be contained, the first step would be to construct the reference square as a function of the proportional modulation of the parallel line where its base rests, such that the height of the trapezium does not surpass the width of the base of the coffer.

2.2 If the first parallel line is taken as the base level of any of the 28 meridian courses, as shown in Figure 3, it can be observed that the outline of the coffer forms a trapezium that can be inscribed in a square whose diagonals coincide, at least in the figure, with those of the inscribed trapezium. It can be shown that the horizontal line that runs through the intersection of the diagonals of the trapezium is equal to the average of the bases of the trapezium. Given that, strictly speaking, the trapezium of the coffer is slightly smaller than the inscribed trapezium, its horizontal line, which is the average of its bases, will be slightly greater than the inscribed trapezium.

2.3 One way of avoiding this small difference would be to use a second square whose outline, according to Figure 2, would be deduced by drawing the first square with its diagonals, as described in item two above, defining the height of the trapezium of the coffer and transporting it horizontally to the height of the intersection of the diagonals, such that the intersection of this horizontal line and the meridian lines defines the construction of the second square.

2.4 Both procedures are very similar. In fact, a third procedure could be proposed if the exact measurement of a meridian course were available. However, our numerical calculation for a meridian course representing the values obtained from an orthographic projection [5], revealed a trend of proportionality in the five levels that correspond to the coffered ceiling. Thus, Figures 2 and 3 can be considered to be an acceptable hypothesis of approximation.

3 Perspective Outline

Since our objective is to find a method for simplifying the perspective outline, we apply the method of the diagonal vanishing point, which, given that the geometric figure in question is a square into which the circles of the parallel courses can be successively inscribed, is also compatible with the method of the distance vanishing point, because both vanishing points, meet at the same point on the visual horizon. These two Renaissance methods of the diagonal vanishing point and the distance vanishing point are applied here under the author's Modular Perspective method. As shown below, the exact outline of the coffers is generated by iterating the same procedure, regardless of their position in the vault.

3.1 The five intervals of the parallel circles are drawn in a horizontal projection, and these in turn are divided into 28 meridian intervals, as shown in **Figure 4**. Since the outline procedure consists in the successive construction of each parallel circle, it is suitable to deduce for each one the stencils that determine the position of the meridian lines, so that, in order to execute the outline of all the parallel circles, this step of the procedure must be repeated six times.

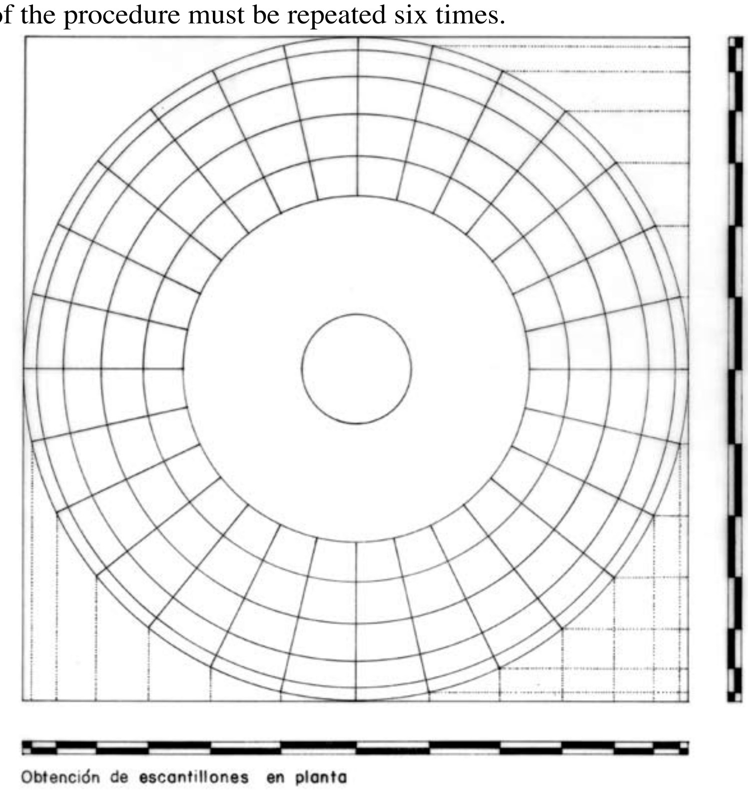

Figure 4. Plan of the parallel circles and the 28 meridian intervals by TGS.

3.2 The section of the building is drawn with the observer located at a distance that is slightly shorter than the radius of the vault, that is, with an angle of observation that is a little over 90°, as shown in **Figure 5**. Subsequently, the height of the observer on which the visual horizon will run is determined. The visual horizon contains the vanishing point and the position of the symmetry visual is fixed at the center of the scene.

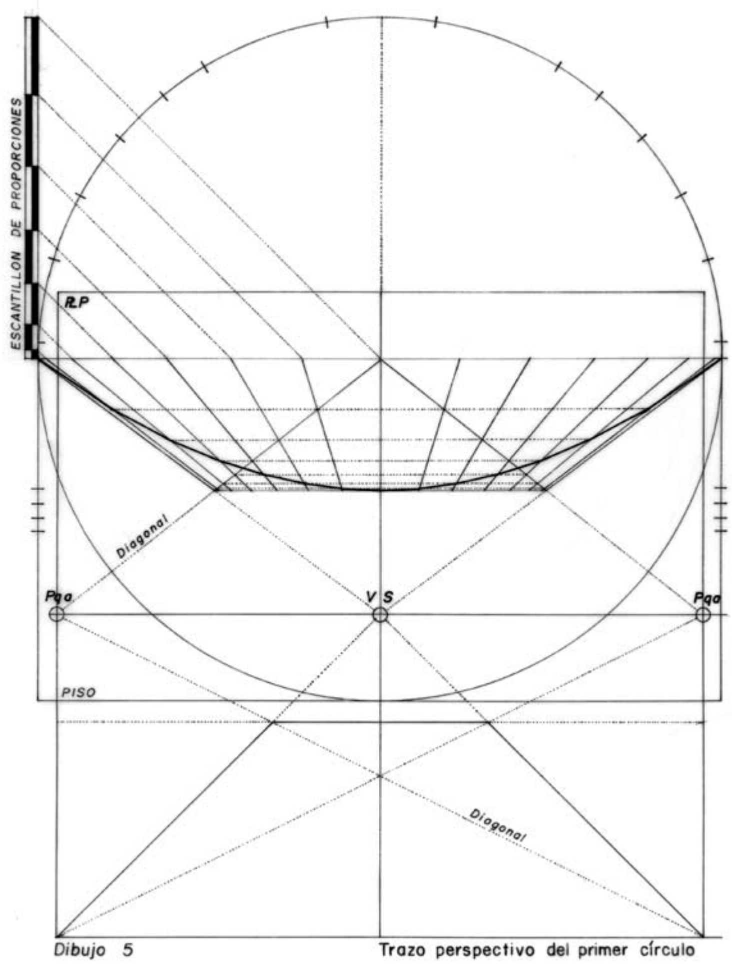

Figure 5. The deduction of the modular depths is obtained be means of the diagonal vanishing point. This application is a variant of the author's method: *Modular Perspective*.

3.3 The distance vanishing point is determined on the visual horizon precisely above the limit of the perspective plane and, by symmetry, this vanishing point can also be represented on the left-hand side of the visual horizon. Since, as mentioned above, in this case the distance vanishing point coincides with the diagonal vanishing point, this vanishing point is used to deduce the depths of the meridian circles to be obtained on each of the parallel circles.

3.4 The height of each parallel circle is marked in the schematic section of Figure 5. The first of these marks coincides with the height of the great cornice which, in turn, defines the base of the vault. As can be observed in this figure, only half of the first parallel circle is shown, since the intention is to construct the perspective of half of the vault, as if the Pantheon were cut in half.

3.5 In Figure 4, in the margins of the plan, are marked some stencils which represent the modular divisions of the meridian circles. The outline of these stencils is repeated for each parallel circle, since, by being concentric, the modulation of the meridian circles is variable. The figure only illustrates the first circle, with the understanding that the outline of the other five circles is generated identically.

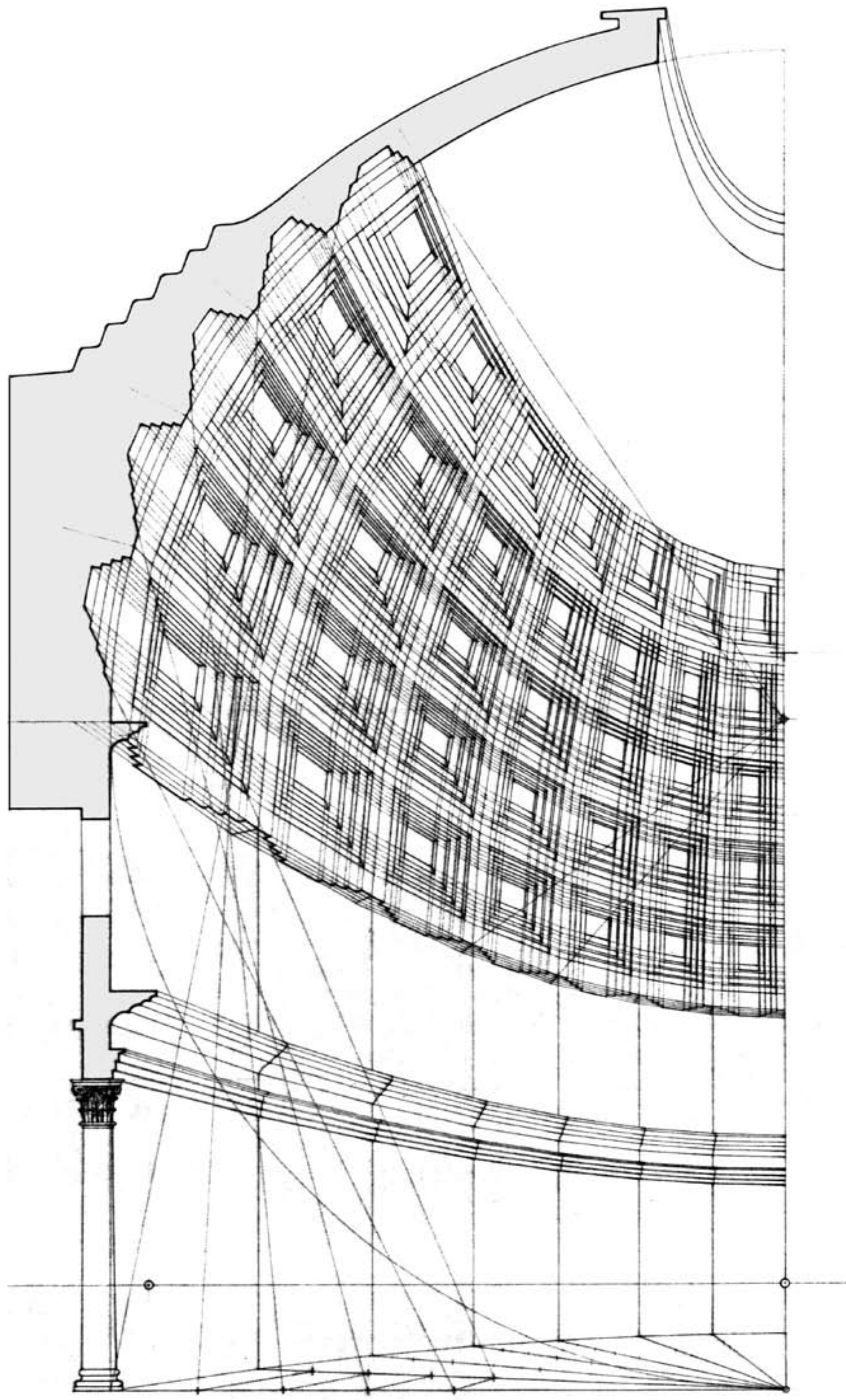

Figure 6. Perspective of the left half of the perspective plane by the author and Jesús M. E.

3.6 The modulation of the meridian circles in the perspective plane is obtained by transporting the stencil of Figure 4 to Figure 5. For clarity, the stencil is positioned vertically at the level of the first parallel circle. In practice, this procedure is simplified by marking the modulations directly on the horizontal line of the plane that contains the parallel semicircle. This horizontal plane represents half of the square that contains the semicircle whose perspective is to be represented, thus, the perspective is obtained exactly by means of the orthogonal conjugation of the stencils, as shown in Figure 5. In this figure, it is important to note that the deduction of the modular depths of the stencil is obtained be means of the diagonal vanishing point, whose principle guarantees the correct generation in depth of the modular values given in the perspective plane.

3.7 This procedure is repeated until the outline of the parallel semicircles is complete. Subsequently, the entire outline of the modulation of the vault, forming the hemispherical reticle of all the coffers, is made.

3.8 The detailed perspective of the coffered ceiling is obtained from the left half of the perspective plane, given that, by symmetry, the lowering of this projection produces the entire outline. **Figure 6** provides details of the constructive section of the coffered ceilings; the outline of the superior and inferior soffit slopes is marked on the profile, while the vertical soffits of all the meridian intervals are marked on the outline of the first parallel semicircle. With these two simple references, that is, by intersecting in perspective the parallel outlines with the meridian outlines, it is possible to generate the perspective deduction of the coffered ceiling. Since this procedure is carried out by means of perspective outlines, it is referred to as a direct deduction procedure. If the deduction were made by relating plan and elevation, it would be necessary to carry out at least six horizontal cross-sections, with the disadvantage that the dimensions of the cross-sections of the coffered ceiling would be incorrect due to their angular variation with respect to the vault profile.

3.9 Finally, **Figure 7** shows the perspective of the vault as a geometric discourse whose central theme, the coffer, moves only in two directions on its hemispherical surface, to give formal expression to its sense of composition. Such formal expression is supported by the pre-Vitruvian principle of the unity of the parts of a whole and the constructive truth of the form. Just as the orifices (or windows) of the skull of the allosaurus lessened the weight of that dinosaur's head, thus avoiding an imbalance in the animal's weight, while allowing for large jaws for the purpose of devouring, the coffers of the vault bear witness to the constructive truth of its great mass, which exists more for the sake of appearance than out of a need to "survive". Our essay emphasized the form and construction of the vault over its perspective representation, since, ultimately, solving its outline merely requires applying the principles of perspective with a certain degree of mastery. However, in architecture, a study of the origin of form, such as that presented here, leads to a better understanding of the generation of the geometry of the form in space.

Abstract (*upcoming article of the author*)

The Pantheon of Rome still conceals a constructive riddle that seems to have no logical explanation, that is, what reason could have motivated the change of the octagonal layout of the ground plan to an heptagonal layout for the vault. One would expect to see a vault of 32-meridians, like San Paolo in Naples, neatly subdivided into fourths from its octagonal ground plan. Instead, the octagonal layout of the ground plan of the Pantheon rises up to the great cornice, and then changes to an unexpected subdivision of 28-meridians. This is a fact, not a theory, as usually we have to deal with in architecture. Even more, the use of two superimposed geometries within a same building was not common practice at the time the Pantheon was built. Presumably, no other building comparable in size with the Pantheon was built at the time, which is why its coffered vault is unique. Standing in the interior of the great aula, one can perceive how the vault's meridians do not correspond with the

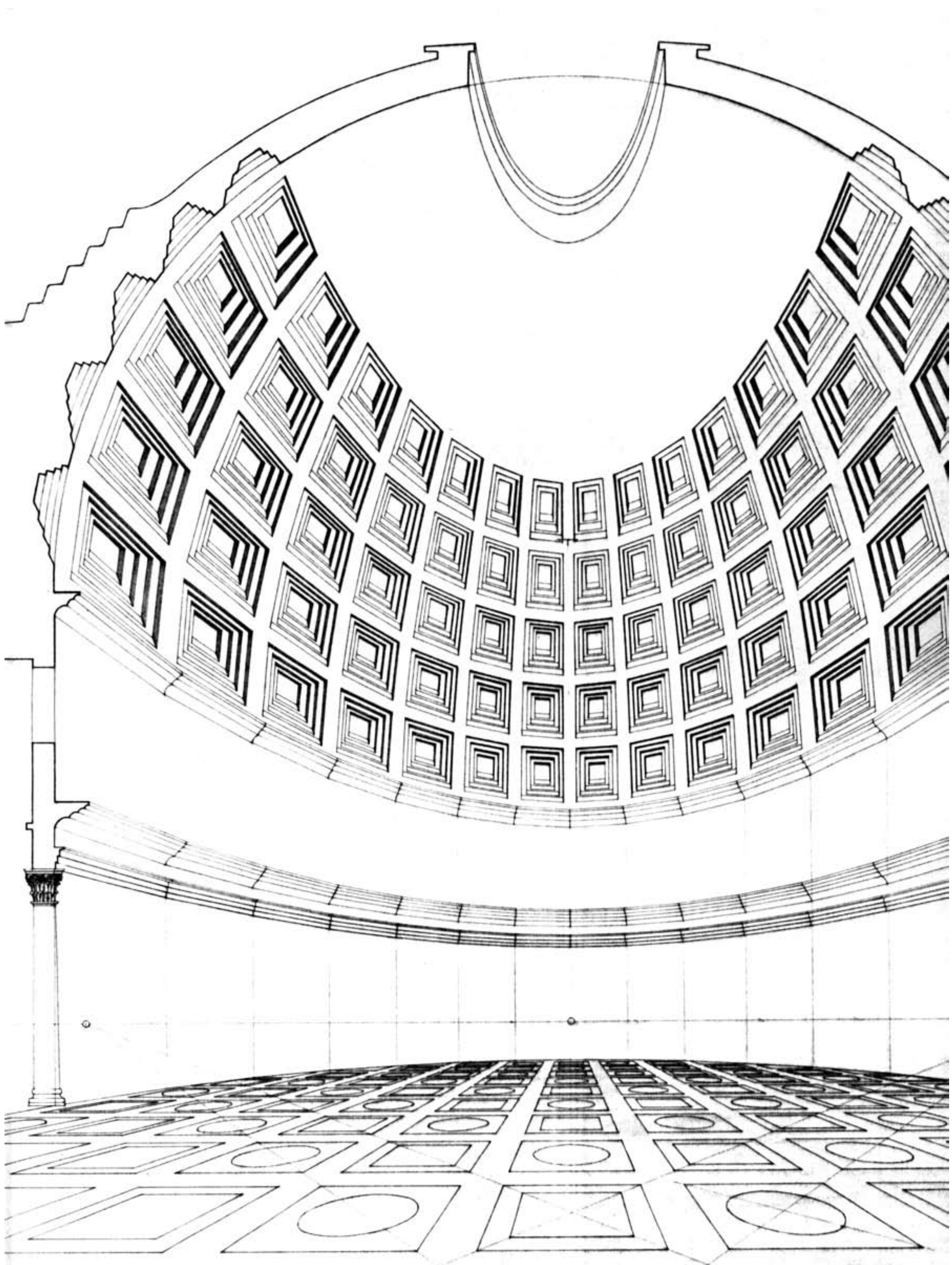

Figure 7. Perspective of the vault by the author and Jesús M. E.

rhythmical distribution of the elements below the great cornice. It takes time to find out where the octagonal and heptagonal elements meet at common points. Moreover, the original fictitious windows all around the attic struggle in vain with the dominant modulation of the coffered vault. We have to wonder thoughtfully what reason might have had the builders to use different geometries; one for the ground plan and another for the vault. On one hand, we can assume that the builders were conscious of the faulty interior they were about to build, and on the other hand, maybe with the intention of preventing the structure from falling down they forsaken all aesthetical precepts. It would be quite ambitious to find out the "why" of the question, in spite of which, the building itself is the only clue we have to find out at least the "how". Being the "how" the central issue of the present paper, we will present here a hypothesis of how the geometrical layout of heptagonal vault was set in place. A hypothesis formulated in accordance with constructive principles, beyond any aesthetical speculation.

Notes and References

[1] W. L. MacDonald, *The Pantheon* (USA: Harvard Univ. Press, 1976), pp. 30-31.

[2] Henri Stierlin, *The Roman Empire* (Italy: Taschen Verlag GmBH, 1996), pp. 158-159 (photo 1).

[3] Piero della Francesca, *De prospetiva Pingendi*, Edizione critica a cura di G. Nicco-Fasola (Firenze: Casa Editrice Le Lettere, 1984).

[4] Even if there had been alternate approaches for envelopment, other than the use of a square, some rule of proportionality must have been defined. Even before Vitruvius, the principle of proportions governed architectural composition, elements and decoration (sense of unity).

[5] Based on the fact that the width of the vault is 43.28 m, the meridian lines would measure 33.99 m from the cornice to the center of the oculus. From the cornice to the horizontal axes of the coffered ceilings the distances would be 5.67 m, 4.53 m, 3.97 m, 3.59 m, and 3.21 m for the five rows, 8.50 m for the smooth surface, and 4.53 m for the oculus. An estimate of the distances between coffers is, in the same order, 0.91 m, 4.38 m, 0.76 m, 3.82 m, 0.66 m, 3.35 m, 0.57 m, 3.07 m, 0.47 m, 2.88 m, 0.19 m, for the five rows. The widths of the lines parallel to the axes are 4.85 m, 4.69 m, 4.33 m, 3.85 m, 3.31 m, 2.73 m, and the widths of the coffers are 3.88 - 3.77 m, 3.74 - 3.49 m, 3.43 - 3.11 m, 3.05 - 2.68 m, 2.62 - 2.22 m. The orthographic interpretation of these values is consistent, since the lines of the outline tend to be continuous. The values will be reviewed at a later date, since an exact measurement or, at least, a photometric substitution study, is essential to minimize the margin of error.

Credits

Photograph 1, Henri Stierlin, see reference [2].
Fig. 1 Andrea Palladio, Pantheon's Section from I Quattro Libri dell'Architettura, LVII.
Fig. 2 Geometrical analisys of the coffered ceilings by TGS.
Fig. 3 The outline principle to generate the woodwork, hypothesis by TGS.
Fig. 4 Plan of the parallel circles and the 28 meridian intervals by TGS.
Fig. 5 The deduction of the modular depths is obtained be means of the diagonal vanishing point. This application is a variant of the author's method: *Modular Perspective*.
Fig. 6 Perspective of the left half of the perspective plane by the author and Jesús M.
Fig. 7 Perspective of the vault by the author and Jesús M. E.

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Aslo it is referred in: http://www.generativeart.com/salgado/salgado.htm

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