THE HEPTAGONAL LAYOUT OF THE PANTHEON'S VAULT

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ABSTRACT

Why did the architect of the Pantheon not carry the octagonal floor plan up to the vault? Why did it have to be heptagonal? This unexpected shift between the floor plan and the vault's layout seems like a decision made by constructive reasons instead of on a whim. Let us start with a simple analysis of what we have at plain sight when looking at the Pantheon's Vault. As many of us have experienced, the most striking feature of the vault is the slanting shape that its coffers' recesses have, making them visible from any place we stand under the vault. While observing said view our first question arises: could a vault of 32-rows of coffers produce a similar effect? Yes, most likely it could, since the beauty of the vault does not depend on the number of rows of coffers that it has. There must be another reason that explains the use of different layouts; maybe a constructive reason that we have not yet noticed. Ironically, the coffered vault itself turns out to be the main clue in this case. Just think about this: how much weight could be saved by designing the vault with 28-rows of coffers instead of 32-rows of coffers. When building, as we know, a slight overweight can be the difference between success and failure, which in every aspect could be a legitimate reason to have made the vault heptagonal. Based on the evidence that we still have, which is the vault itself, we will here present two hypotheses about its heptagonal layout. To my knowledge, these hypotheses have not been stated or discussed before.

1- INTRODUCTION

The construction of the heptagon by straightedge and compass was among the four unsolved problems of antiquity.[1] In fact, it remains unsolved even at present. To overcome this problem the Greeks invented marked rulers, a method they called *neusis*.[2] A practical method which Oenopides (ca. 450 BC) did not favor since he believed that compass and straightedge constructions were superior to *neusis*. Moreover, in my opinion, *neusis* is not a legitimate method of proof because it does not consider step by step the elements of the constructive process upon which the proof itself has to be built.

Could the architect have invented a method to outline the heptagonal vault far from the so-called "marked rules? Most likely, yes, if he were compelled due to a constructive necessity. At the time some structural problems, such as the weight of a structure, were only estimated by practice and not by exact numerical calculations. As we will see later, practice has been the best teacher for builders.

1 The four unsolved problems by ruler and compass from antiquity are the doubling of the cube, the angle trisection, squaring a circle, and the construction of a perfect heptagon.

2 *Neusis* (*neuseis*/ plural) is a geometric construction method based on marked rulers invented by the Greeks.

What if the architect realized that extracting the weight of 28-meridians of coffers, instead of the weight of 32-meridians of coffers, would make the vault lighter? After all, the golden principle of large vaults, which is to achieve the maximum span with the least weight, comes from Roman architects. We then will discuss the subject matter of the vault's weight.

Accurate drawings of the Pantheon's vault produced with in situ measurements taken with high-tech instruments seem to not have been made yet. Of this kind, the study of the interior of Hagia Sophia that Volker Hoffmann [2005] carried out in situ is remarkable. With the aid of a new Leica-Scanner Cyrax 2500 he obtained the necessary data for the development of accurate drawings. He then was able to prove his analemma hypothesis for both the ground plan and the internal facades of the basilica. In our case, there is no need of an accurate drawing of the vault since our goal is to approach a hypothesis of its geometric layout. On the other hand, as far as I know, there are not any extant original drawings, nor any drawings prepared in situ, of the Pantheon's vault,[3] such as those remarkably discovered by Lothar Haselberger in 1979 in the form of tiny lines incised into the marble of the high-interior walls of the temple of Apollo (Didyma, Turkey), which depict in full-size the original outline of the temple's columns.[4]

2- Constructive Principle of the Dome

The Pantheon[5] vault has to support its own weight (dead loads), wind load and snow (live loads). The main structural elements of the vault are: the meridians (virtually crossing at the oculus' center),[6] the parallel hoops (which impose structural continuity along with the meridians), the coffers (to reduce its weight), and the oculus (acting as a compression ring). The thickness of the vault section decreases as it rises, from 6.5 m at its base to 1.5 m at the top; a rational section to withstand lateral thrusts and so prevent its possible collapse. Thus, the vault takes both the tensile stresses by the parallels and the compression stresses by its upper plain surface. According to Salvadori [1990: 226], the parallel unchanged lies at 52º in a hemispherical vault. This is found in the Pantheon at the last ring of coffers with an angle of 55º or so.

To be precise, what makes stable the dome's structure is the coaction of both meridians and parallels. As we know, the extracted weight of the coffers is little in comparison to the vault's total weight; however, this guarantees the structural safety of the vault. To get an idea of how heavy the dome is, just picture that its annular base has an area of 1,019 m2. Thus, the weight of the vault at 2.7 m height is nearly 5,000 t, which is the weight that some sources hold as the total weight. Now, using our theoretical model of the dome,[7] with a uniform section of 1.5 m and radius of 21.7 m,[8]

3 Roman architects were familiar with drawing and models even before Vitruvio's *De Architectura*. On the subject, see Rabun M. Taylor. *Roman Builders* (UK: Cambridge University Press, 2003), 27-36.

4 Lothar Haselberger. "The Construction Plans for the Temple of Apollo at Didyma," Scientific American, 253, No. 6, (1985), 126-132.

5 Marcus Vipsanius Agrippa is the author of the original design and construction of the Pantheon of Rome (27-25 BC). After a fire severely damaged the structure, Hadrian undertook its reconstruction, modifying probably its original design (120-125 AD). Subsequently, Septimus Severus maintained and restored the building (183-211 AD).

6 From here on the term 'meridian' is use instead of 'radial row of coffers.' Although a radial row of coffers actually is halve-meridian of coffers.

7 My assistant, Daniela Franco, and I, developed a 3D-model of the Pantheon to estimate the volume of the vault, and so that of the coffers.

8 Most sources refer a span of 142 feet across (43.2816 m), or 142.4 feet (43.4 m); which is the datum here it yields a volume of 4,752 m3. Therefore, if the dome were made of lightweight concrete (1,350 kg/m3), it would weigh 6,415 t; whereas, if the dome were made of heavy concrete (2,200 kg/m3), it would weigh 10,454 t.[9] Which is why, to keep the vault's weight in check, the builder gradually placed the aggregates from heavy to lighter; to end with tufa and pumice at the oculus spot.

Evidently, the control of the vault's weight, at any stage of the process, was of paramount importance for the builders.[10] As we stated before, a slight overweight can be the difference between success and collapse when building. Which is why we have to wonder what difference can extracting the weight of 28-meridians of coffers instead of 32 make. Just think about this: what makes the flying buttresses of Gothic cathedrals stable? Amazingly, it is due to the small weight added by the pinnacles.[11] Either way, adding or extracting weight, can improve the safety of a structure. The latter is the Pantheon's vault case, whose coffers reduce its weight to the order of 8-9 %, or around 1,400 t. In fact, if the vault were of 32-meridians, its weight would have been as much as 230 t heavier.

Like in a kid's teeter-totter, just a little weight added at one of its sides can make two heavy weights move to that side. If 28- meridians of coffers can truly reduce a little more the weight of the vault, as much as 230 t, how could the builders have known so? Suppose that they built a mold for each one of the coffers, filling up the smallest with sand and emptying it on the ground 32 times, doing the same with the bigger one 28 times, then it would suffice to compare which of them threw out more sand on the ground to get the right answer.

Although, my numbers come from a theoretical model, they give us an idea of the total weight and that of the weight extracted by the coffers. In other words, we can predict what happens when coffers change in size, that is, to which side of the balance their respective weight would move it. All this suggests that the architect took into account the weight of the materials before building the vault. Therefore, once he became aware that the vault would be lighter by using a 28-meridians partition, he threw away the 'ideal' 32-meridians design; what other more convincing reason could there be than this one? The involvement of aesthetic or symbolical aspects in the vault's design, as some scholars claim, is not convincing. Not because these aspects might be true or false, but because they lack constructive arguments. In my opinion, as an architect, the size of the coffers was mainly determined by their function and later was their aesthetic form considered.[12] After all, the gospel of "form follows function" comes from antiquity.

The original design of the attic strongly suggests a decoration which disguises the lack of

used. Other sources used the Roman foot (0.296 m).

9 Concrete weights estimated according to THOMAS NELSON WINTER. "Roman Concrete: The Ascent, Summit, and Decline of an Art," Transactions of the Nebraska Academy of Sciences —Volume VII, 1979, p. 141. http://digitalcommons.unl.edu/cgi/viewcontent.cgi?article=1000&context=classicsfacpub

10 Not solely the weight of vault was of paramount importance; it also was the safety of the entire structure. Relieving arches around the cylindrical wall helps to reinforce it, which in combination with its many cavities and chambers reduced its weight to maintain an ideal section to support the lateral thrust of the vault.

11 EUGÈNE VIOLLET-LE-DUC. *Dictionnaire raisonné de l'architecture française* (Paris: Librairies-Imprimeries réunies, 1860). He largely discussed the function of both buttresses and pinnacles in Tome 1, and Tome 7. In T.1, he says: "Au-dessus des arcs-boutants les contre-forts sont rendus plus stables par des pinacles…" http://fr.wikisource.org/wiki/Dictionnaire_raisonné_de_l'architecture_française_du_XIe_au_XVIe_siècle

12 Roman builders were aware of the Vitruvian triad: *firmitas*, *utilitas*, *venustas* (beauty).

symmetry between the vault and the aula;[13] a lack of symmetry that the actual decoration further enhances.[14] See figs. 1a, and 1b.

Figures 1a, and 1b. View of the vault and the attic. In 3a, a meridian axis coincides at the center of a blind-window, while in 3b, the center of the coffers lies at the center of another window. Photographs by the author.

3- The Wooden Dome

The main axis of the octagonal plan of the aula runs from North to South at two of its vertices, while the remaining six vertices lie at the center of the niches and at the bisection of each side of the octagon the aedicula takes their place. On a sixteen-side polygon, one would expect to see a vault of 32-meridians, but instead it turns out to be a vault of 28-meridians. What if the 28-meridians besides reducing the weight of the vault could also have made the constructive process of the wooden dome more efficient? The term 'wooden dome' encompasses: all the centering frames, the woodwork of the coffers' molds, and the craftwork to complete the outer surface without leaving empty spaces (see fig. 2). In fact, the dome of the Pantheon was first built out of wood and then in concrete. The wooden dome had to be strong enough to support several times its own weight; a risky undertake since the balance between thrust and weight was at stake all the time. From the question about the extraction of the coffers' weight, we could now similarly ask: what difference could lifting the weight of 28-centering frames instead of 32 make? The time consumed by handcrafting each centering frame of nearly 12 t could have been another crucial factor in favor of the heptagonal form of the vault.

13 At the right side of the apse, one can appreciate a small section of the attic's original design. It depicts sets of three pilasters, alternating with blind-windows, resting on top of a parapet all of them. As the sequence seems to repeat at each 1¾ of a meridian, it would encircle the 28 meridians by carrying it out 16-times. This decoration brings in mind the 'fantastic motifs' frequently used in Roman architecture.

14 Regarding the attic, Marder says: "Sangallo famously «corrected» the lack of vertical congruity between the main order, the orders of the attic, and the ribs of the dome in the original composition — a lack of alignment he called «a most pernicious thing.»" See: Tod A. Marder. *The Pantheon after Antiquity*, p. 147.
http://www.digitalpantheon.ch/Marder2009/Marder2009.pdf

It is very unlikely that the legend of the interior being filled up with soil and scattered gold coins, to ensure its removal as soon as the vault cured and gained its strength, actually took place.[15] The lateral thrusts, from the inside outwards of more than 53,000 m3 of soil would have cracked the round wall. Even the use of a complex framework resting on the floor, to support the wooden dome is doubtful due to the enormous amount of wood required. Instead, the theory of the centering frames fabricated in halves, on the floor, seems plausible.[16] If so, then by using a hoist, each half could have been lifted to the annular base at opposite points of a meridian axis, and once the halves were perfectly aligned and linked together would work as a self-supporting structure. In the same manner, the remaining 13-meridian frames could have been placed, one by one, at the annular base. One can only imagine what the wooden dome might have looked like. It ended up most likely in bonfire and with it, half of the vault's history was lost forever.

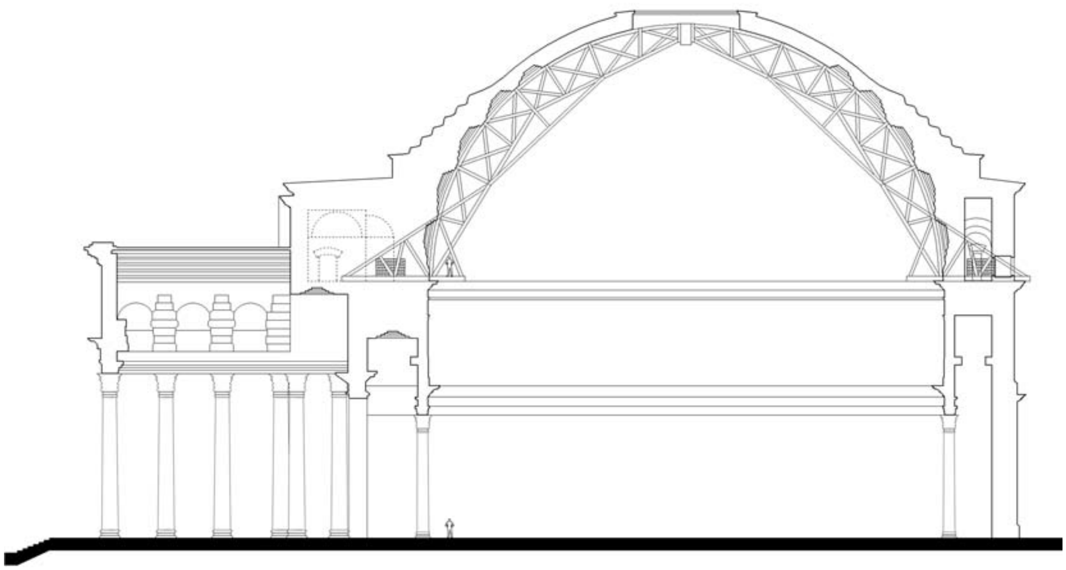


Figure 2. The centering frames interpreted by the author based in Viollet-le-Duc (Tome 9, Figure 4). Drawing by Christian Esparza and the author.

4- The Unsolved Problem of the Heptagon

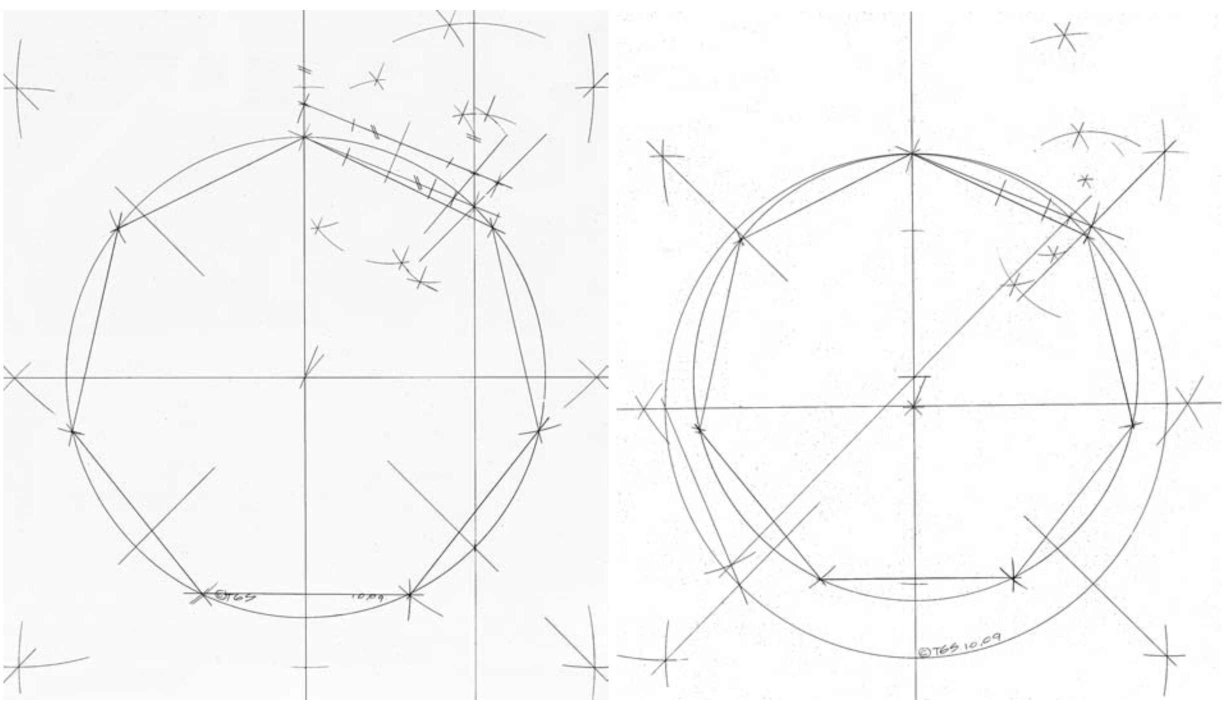
At the time, the construction of the heptagon was a challenging problem to solve either for geometricians or for builders. The Roman builders had their own methods to construct regular polygons, they used them to set a plan on the ground, to design decorative elements, or even to shape symbols in polygons, such as the heptagram (a seven-pointed figure). Although we do not know what their methods were, we can suppose that a large marked ruler could have been used in the case of the heptagonal vault as it was, overall, easy to manipulate around its annular base and needed no compass. However, there could be another method, even better than the use of a

15 A legend attributed to John Capgrave (1333-1464), who probably got the idea, of an interior filled with soil, when he saw the construction of a cellar in St. Thomas Hospital at Rome.

16 See: Eugène Viollet-le-Duc (Tome 9, Voûte, p.11).

large marked ruler. A method that only in the course of practice the architect might have realized if by chance he were playing with a cord, as we will see in section 6. Supposedly, the architect should have known how to outline a heptagon before the cylindrical wall reached the level of the annular base. After all, standing on the annular base was not the right place to start thinking of the problem.

Thinking as if I were on top of the annular base of the vault, about to outline a heptagon within its limits, two constructive methods occurred to me. In the first method, one side of the octagon is divided into 7 parts; while in the second one; one side of the octagon is divided into 8 parts (see figs. 3a, and 3b). Both constructions have their sides thus divided, and no error appears to have been committed. I particularly got the impression that the second construction was correct. Here, when dividing into 4 modules a side of the heptagon, it yields the 28 radii of the inner perimeter of the vault. For a day or so, I convinced myself of having solved the famous problem by compass and straightedge until the numbers proved me otherwise. In fact, what I found, illustrated in fig. 3a, is how the sides of a heptagon match with those of a given octagon when both polygons circumscribe to a same circle. In its turn, in fig. 3b, the numbers do not match when calculating the central angles of the heptagon. Here, the central angles, instead of being of 51.428571º, they resulted of 51.870161º. This tells us that the heptagon side should be a little bit shorter, as much as 0.06345 of a module, to get the central angles accurately. After all, the Gaussian sufficient condition for a regular *n*-gon to be constructible seems insuperable.[17]



Figures 3a and 3b. Two constructive methods of the heptagon by the author. A side of the octagon is divided into 7 equal parts, in 3a, while: a side of the octagon is divided into 8 equal parts, in 3b. Silver-point drawings by the author.

17 “A regular *n*-gon can be constructed with compass and straightedge if *n* is the product of a power of 2 and any number of distinct Fermat primes.” http://en.wikipedia.org/wiki/Constructible_polygon (accessed 04.16. 2010).

These efforts were not in vain. What I have learned is simple but effective. To outline a heptagon on a given circle, we have to find out an arc-segment, of such a length, that carrying it out 7-times along its perimeter will circumscribe it perfectly. After all, both the octagon and the heptagon must fit within the same circle.

In doing this, and using only straight cords to outline the heptagon, an intriguing geometrical coincidence came up; maybe the same one that the architect of the Pantheon could have realized and put in practice (see fig. 4). In other words, what this coincidence tells us is where the sides of an octagon intersect with the sides of a heptagon.[18] Moreover, it also suggests, that even though the ground plan seems to follow an "ideal plan", it is likely that the underlying plan of the vault was envisioned as part of its constructive process. In the end, the underlying plan was mixed along with the masonry of the vault, leaving no trace of it. It is at this point that our story begins.

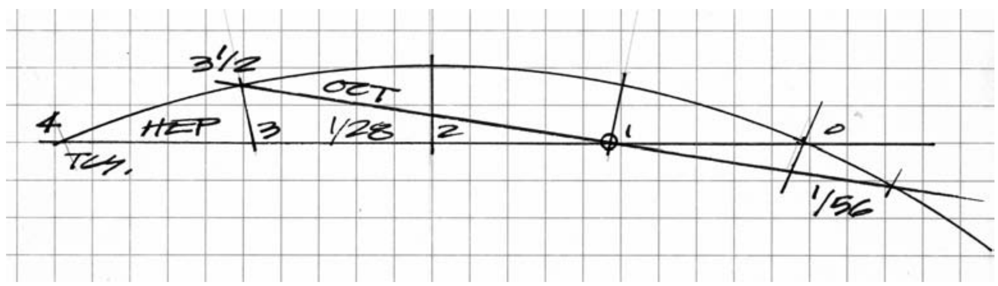


Figure 4. Author's sketch by which the solution of the heptagon construction was found; as simple as: 4 x 7 = 28 = 3.5 x 8.

5- First Hypothesis of the Heptagonal Layout of the Pantheon's Vault / method 1

The annular base was large enough to allow outlining the perimeter of the heptagonal vault on it. In constructive terms, on the workable space of the annular base of 6.5 m, it was feasible to outline on its surface all sides of the octagon as well those of the heptagon. Such a workspace could not be of better dimensions. However, the major problem was rather of procedure: how could the points of intersection between the sides of the heptagon and the sides of the octagon be determined. To carry it out, the architect knew beforehand that there was an unavoidable restriction: the heptagon would have to be deduced from the octagon within the same circle. Such a circle most likely was the outer perimeter of the vault, since the outline of all sides of both the octagon and heptagon would lie on the annular base. This is the key question that supports my hypothesis for the heptagonal layout of the vault. It now follows where to start (see fig. 5).

First, it was necessary to relocate the north-south and the east-west axes on the annular base, a simple task, which consisted in re-locating the four cardinal points. To do this, the cardinal points would have to align vertically to their corresponding positions already set on the ground plan. Sighting devices, such an alidade and plumb lines, could have helped to accomplish this purpose;

18 The point of intersection between a side of the octagon and a side of the heptagon seemed accurate in my drawing, but it is not. There is a discrete deviation (unnoticeable to the naked eye) produced by the thickness of the lines, which, of course, linear equations can estimate accurately.

marking the axes on the annulus by chiseled lines afterwards. At this stage of the process, the great cornice could have been placed at the rim of the annulus, after carving each one of its segments.

Then, the layout of both the inner and outer circle on the annular base follows, conformed along with the chiseled lines the initial layout of the vault. It sounds easy, but the outlining of both circles, without the aid of a fixed center, was not that easy. The drum wall itself could have been useful to carry out the outlining by sliding a sort of square-rule around its internal rim.

Now, to trace the octagon on the outer circle, it was also necessary to relocate the middle points between cardinal points, in the same way in which the cardinal points were set on the annular base. That is, by using plumb lines to verify its position from the ground. The architect then verified that all sides of the octagon were of equal length, in order to trace its perimeter on the annular base.

Thus, standing at north point of the annular base, by joining the north and northeast points, one side of the octagon is determined. Applying the same method, but now on the right side, allows us to determine another side of the octagon, and so on until we find out the remaining sides

Now comes the magic solution. If we divide the arc of a side of the octagon into three and a half sub-arcs, it would result that we have 3.5 x 8 = 28 meridians. This means that a sub-arc, of 1/56 of the perimeter, is common for both the octagon and the heptagon, hence, by carrying 1/56 at both sides of north point, a meridian on the annulus would be laid. The same procedure is true for the other three cardinal points.

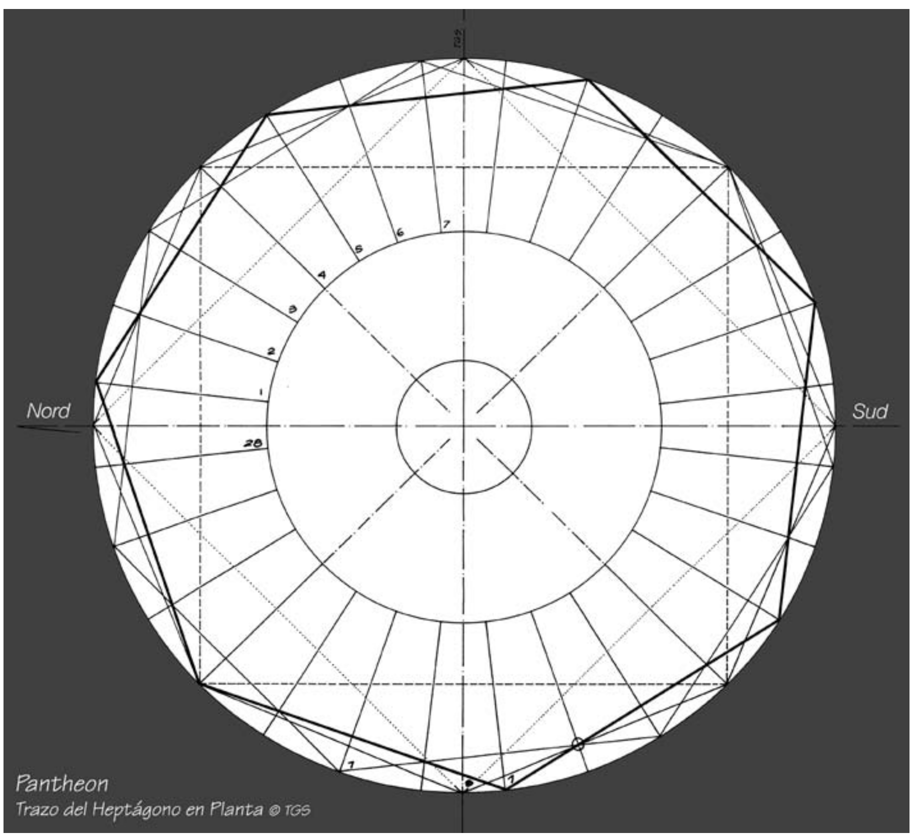


Figure 5. Hypothesis for the Pantheon's vault outlining according to the author. Drawing by the author.

Only a 3.5-radius partition of an octagon's side holds true for a heptagon when both circumscribe on the same circle. There is no other way to get the center of the meridians (not the center of the ribs) to coincide at the four cardinal points. Otherwise, if the center of the ribs were in coincidence with the cardinal points, the resulting appearance would have been disastrous. It was wise to make both the center of the meridians and the cardinal axes coincide, a design that in a way disguises the lack of symmetry between the vault and the niches. In regard of the attic, see notes 13 and 14.

In consequence, the operation {3.5 x 8 = 28} = {4 x 7 = 28}, means that each side of the heptagon would correspond to each segment of four sub-arcs. Here, the numbers 3.5 and 4 denote sub-arcs, while the numbers 7 and 8, denote sides. Therefore, if a 'segment' is carried out 7-times on the circle, it will generate a perfect heptagon. Fig. 5 shows how the perimeter of the inner circle of the vault (136.345 m), the length of the heptagon's sides (18.8306 m), the length of the sub-arcs (4.869464 m) and the central angles (51.428571º) all fit accurately.

Figure 6a. The reticulum of the floor, about 5 x 5m, serves as scale of reference.

6- SECOND HYPOTHESIS OF THE HEPTAGONAL LAYOUT OF THE PANTHEON'S VAULT/ METHOD 2

Method 1 —elegant for geometers— raised an intriguing question: how could the length of a cord be divided into 7 equal parts manually and not by metric methods? Method 2, as we will next see, offers an ingenious solution to this riddle. This is how method 2 goes: we use a cord of the length of the perimeter of the vault, joining its ends and dividing it in half, do the same again to divide it into fourths, and once more to divide it into eighths. Then, we insert two bars through all the cords at their looped ends. Then, pulling steadily on the bars, as to have 8 equal lengths of rope, we let go one end of the rope. Now we slowly pull the poles away from each other, until the loose end, which we just let go, reaches the opposite pole, leaving seven equal lengths of rope, each for one side of the heptagon.

When my son Tomás saw me with a short cord folded into 8 parts, trying to unfold it into 7, he told me: just loose one end and pull them apart. Easy does it. We effortlessly did it with a short cord, but the next question was if it would be that easy when using a full-sized cord? There was only one way to find out: to carry out a one-to-one experiment. For the experiment, we (my students and I) just needed two things: a cord of 136.345 m in length, and 100 arc segments (of 53.6 x 3 x 3 inches) made of cardboard to replicate the perimeter of the vault's base. See fig. 6a.

Once ready to carry out the experiment in campus (UNAM, May 5, 2010), our first action was to draw on the ground a circle with a radius of 43.40 m. We then laid on the ground and attached side-by-side 100 arc segments made of cardboard conforming a circle, and then passed the cord around the circle to measure the length of its perimeter. Next, we removed the cord and proceed to divide it into 7 equal parts, using the method we mentioned above.

The rest was simple; first, we verified the heptagonal partition, then by dividing each one-seventh length into 4 equal parts, we marked out the meridians' partition on the ring. The latter was the easiest step; we just folded one seventh twice and it was done. See fig. 6b.

In less than 4 hours, we found ourselves walking around the layout of the annular base of the Pantheon's vault. The Pantheon's architect could have easily applied this method by laying a ring made of mortar and bricks on the annular base. Actually, I do not see how it could be as simple and effective as turns out to be in method 2.

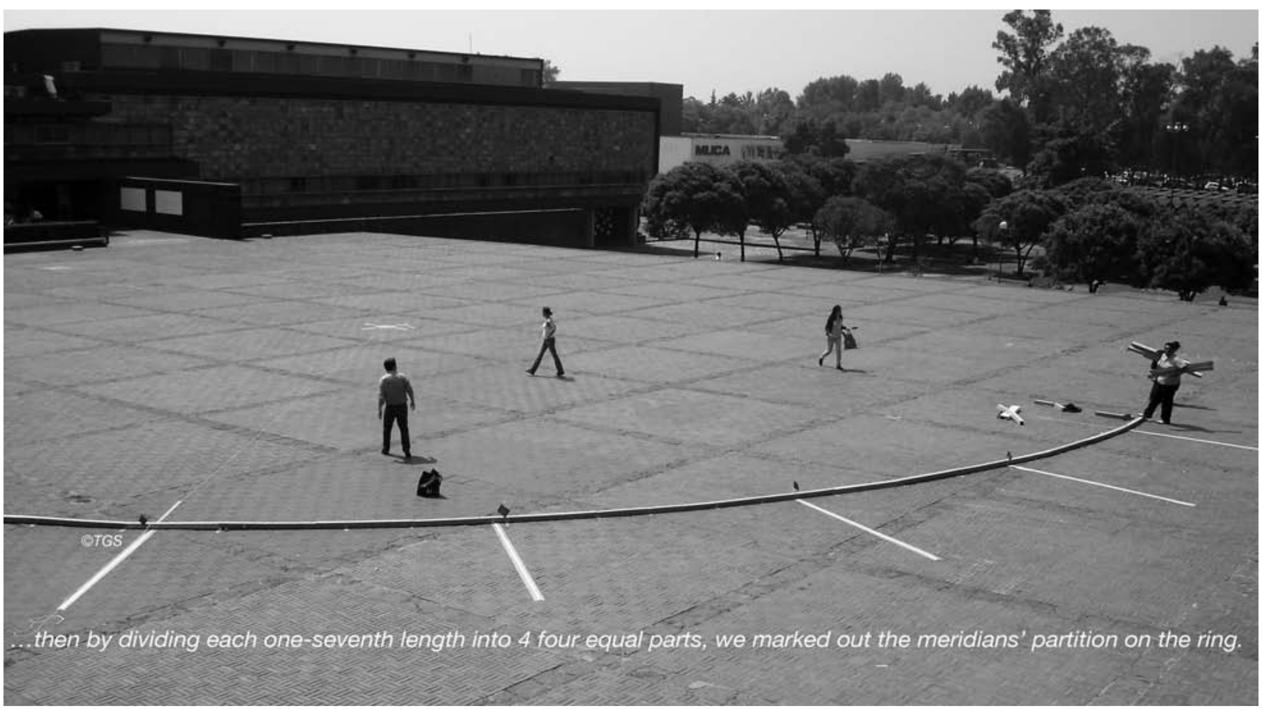


Figure 6b. The radial alignment of the meridians was controlled from opposite sides, as it could have been controlled in reality. Photograph by Ricardo Silva Portillo.

7- The Slanting Recesses of the Coffers/ Perspective or Distortion

I have already discussed, in a previous article, the form that the coffers have in frontal view [García-Salgado, 2009]. Now, I would like to briefly refer to its sui generis form in profile. The lower recesses of the coffers slant at a constant angle of ± 20º along the curvature of the meridians while the upper recesses slant the same way but at ± 90º. The enlargement of the lower recesses of the coffers increases its sunken effect, and at the same time, keeps them in sight (see fig. 7). That is, that according to their size the coffers become distorted along the meridians if we observed them in profile; in the frontal view, they maintain their diagonals constantly at an angle of ± 45º (see fig. 8).

A simple principle, but effective; was to slide up the sunken faces of the coffers, as to make their lower recesses visible from the floor. From the constructive point of view, the coffers' profile with its lower recesses slanting downwards would have helped the pulling out of the centering frames without cracking the concrete. I have surveyed the vault for hours, from the floor, without succeeding in finding out where the vantage point may be. When one looks at the coffered vault, from anywhere on the floor, one does not perceive a perspective effect, but distortion. Because of the ever-changing size and shape of the coffers all over the inner face of the vault, it would be logical to think of the center of the floor as the vantage point; but from here, only a constant distortion of the coffers can be perceived, without producing any perspective effect (see fig. 9).

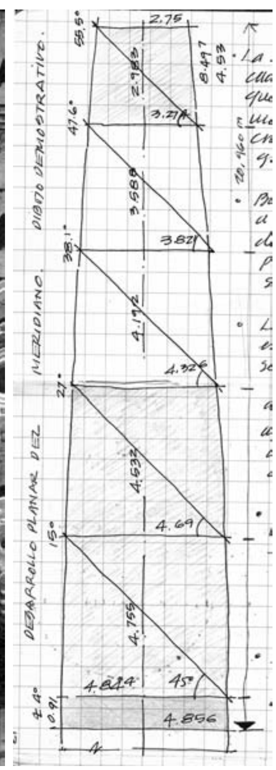


Figure 7. View of the vault. This photo was taken out of the center of the floor, and yet all the recesses are in view. Photograph by the author.

Figure 8. Sketch of a row of coffers. Drawing by the author.

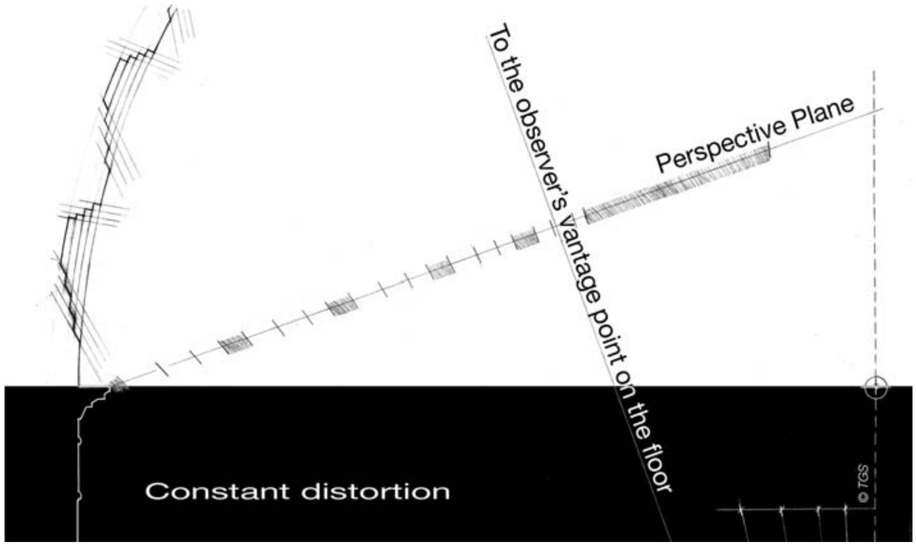


Figure 9. Visual projection of the coffers in constant distortion. Drawing by the author.

Perhaps, it would be better say that a non-specific “image formation” occurs when viewing the coffered ceiling. This concept, image formation, can help us to understand why all the coffers seem to accommodate to our sight as we walk on the floor. In fig. 10 it is evident how none the lower recesses vanish at any one point in particular; but rather at many points.

Conclusions

No records about the constructive process of the vault have survived. Nor a sketch regarding the ephemeral layout carried out on its annular base and the real one ended up engulfed by the poured concrete. In spite of this, I have proposed here two attainable methods for the outlining of the vault, which at the very least put forward the complexity of the problem. Most importantly, what the real-size experiment of method 2 makes evident, beyond its own feasibility or beyond what the true method might be; is that, then and now, practice can amaze theory. Method 1 took me a year, if not more, while method 2 took me minutes. Most likely, the design of the vault was ruled by constructive principles instead of the sake of pure form or symbolic meanings. After looking into all the details and having learned that the vault would be lighter if the coffers were bigger, the architect could have anticipated that the heptagonal form would be more efficient than the ideal one. In my opinion, by all means, the aim of the architect was to reduce the weight of the vault in order to get along with the king of all forces: gravity. In every sense, the heptagonal vault turns out to be the most intriguing and elegant element of the Pantheon; it attracts our sight while making us wonder, how can it be there without falling down. Not less remarkable was the brilliant idea of the slanted coffers that make the vault look as if our own hand could touch it. Maybe such a magnificent view of the Pantheon's vault motivated Cassius Dio to say: “… it resembles the heaven.”[19] And who has not attempted, in a starry night, to ‘touch’ the heavens.

19 “Also he completed the building called the Pantheon. It has this name, perhaps because it received among the images which decorated it the statues of many gods, including Mars and Venus; but my own opinion of the name is that, because of its vaulted roof, it resembles the heavens.” Cassius Dio. *Roman History*, Book LIII, p. 265. http://penelope.uchicago.edu/Thayer/E/Roman/Texts/Cassius_Dio/home.html

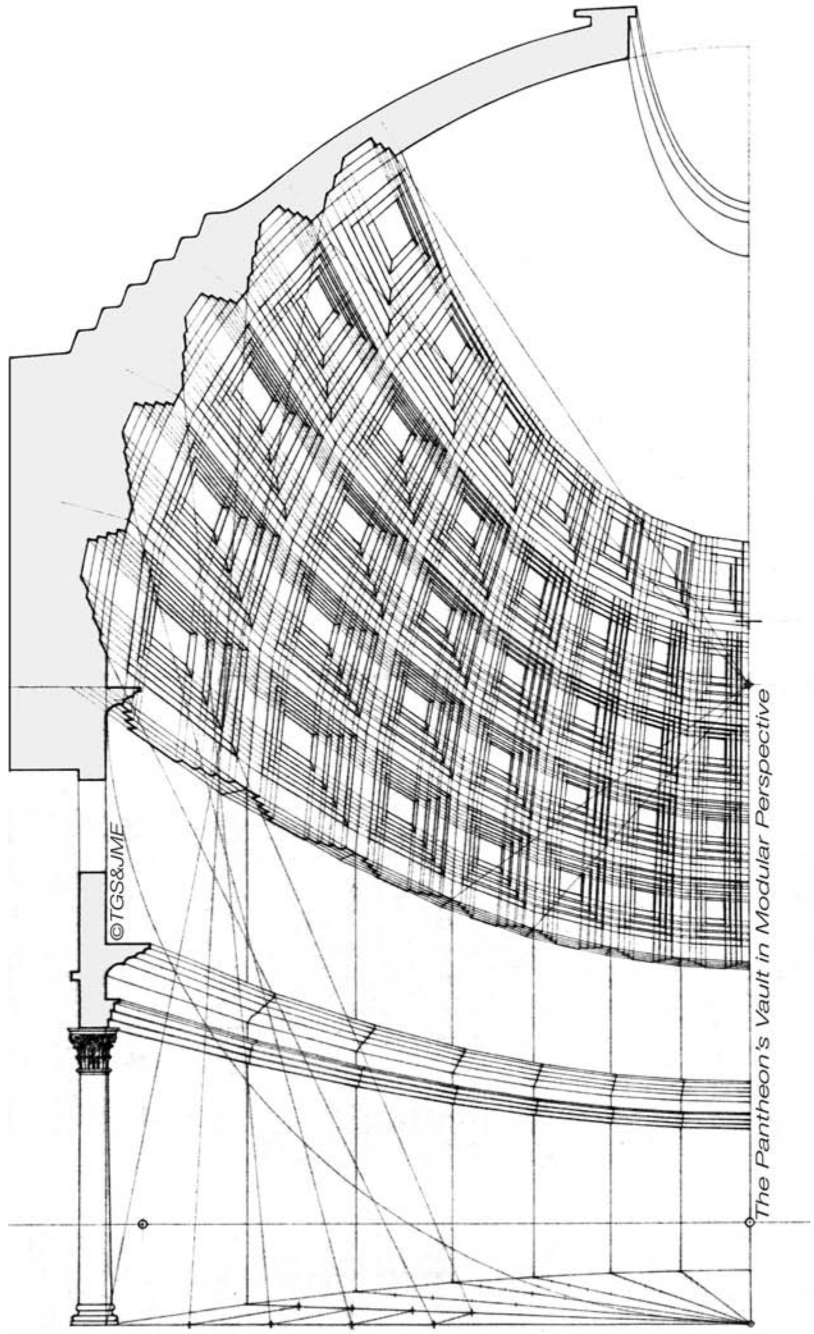


Figure 10. Cross section of the Pantheon's vault in Modular Perspective. In this drawing, it is clear that lower recesses of the coffers vanish to different points on the floor. Drawing by Jesús Manzanares and the author.

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