

# THE PERSPECTIVE *of* LEONARDO'S LAST SUPPER

**Dr. Tomás García-Salgado**

Faculty of Architecture/ UNAM/ SNI-III

Website <[www.perspectivegeometry.com](http://www.perspectivegeometry.com)>

## Index

1 Abstract. 2 Premise. 3 The *Costruzione Legittima*. 4 On the Perspective Outline. 5 Constant Distance. 6 Variable Distance. 7 Natural Distance Vs. Focal Distance. 8 Leonardo's Experiment. 9 Conclusions: Hypothesis of the Perspective Outline, Visual Experiment. 10 Bibliography And Notes. 11 Glossary.

## 1 ABSTRACT

My approach to the science of perspective is theoretical and practical because my interest in this science centers on the invention of a new method and its application basically to architectural perspective. The first part of this paper discusses the construction of perspective in Leonardo Da Vinci's *Last Supper* and, as a conclusion, a hypothesis on the outline for the reconstruction of the refectory is offered. However, the historical and critical analysis of this highly significant and unique painting is essential to situating my subject matter in its time and circumstance, and therefore the second part of this paper is about the various interpretations other authors have made of this work. The two parts of this paper may be read separately. The perspective drawings, which illustrate this paper, were drawn according to Alberti's *costruzione legittima* as interpreted by Leonardo, and according to the Modular Perspective Method developed by the author [1].

## 2 PREMISE

The fundamental interest of authors such as Veltman (1986), Pedretti (1973), Polzer (1980), Naumann (1979), Steinberg (1973), Hoerth (1907). Möller (1952), Bossi (1810), White (1957), Hartt (1969), Wright (1983), Kubovy (1986), and others, centers, in one form or another, on trying to reconstruct the architecture of Leonardo's illusory refectory, which, luckily, is a part of the composition least altered by the restorations this mural has undergone. It is this same interest that I share here with my readers.

In order to analyze the perspective used in the *Last Supper*, one must ask how the scene was conceived, how the outline was drawn, and how far from the fresco Leonardo placed himself. The only effective evidence available is the work itself, which is defined by the two basic elements of its composition: the Savior with his apostles, and the architecture of the refectory.

The initial questions lead to the fundamental question, which is related to the drawing of the vantage point of this work, of how Leonardo applied the *costruzione legittima* to draw the lines of perspective. Since there is no historical evidence, such as a sketch of the refectory, to help answer this question, I offer the hypothesis that Leonardo knew how to draw the vantage (*distantia*) point, measured along the horizon line (*linea centrica*), starting from the vanishing point of the orthogonal lines (*punto centrico*) and terminating at the vanishing point of the proof diagonal. This hypothesis is based on considerations of a theoretical and practical nature [2] which afford an analysis of certain ideas relating to this work that have not been discussed previously, such as the concepts of *focal distance* and *natural distance*. These considerations may be controversial for those historians versed in Renaissance perspective. However, an attempt to analyze this work solely with the historical evidence would be limited, since both Leonardo's manuscript and originals on perspective were stolen [3].

The information available is basically contained in Manuscript A (1492). It is uncertain whether Leonardo wrote all he knew, including technical details arising from solutions to given problems. Leonardo's *Trattato della Pittura* is a transcription of his notes, which has been reproduced many times. One of such copies belonged to Benvenuto Cellini and, according to Clark, might have included Leonardo's treatise on perspective.

### 3 THE COSTRUZIONE LEGITTIMA

According to Naumann's [4] well-documented references, Leonardo was familiar with Euclid's *Optica* (300 B.C.) and Witelo's *Perspectiva* (1270), and he probably had knowledge of Roger Bacon's *Opus Majus* and John Peckham's *Perspectiva Communis* (1270), which were works based on the theory of light beams and the concept of a visual cone in whose vertex is positioned the eye of the observer. These works discuss the phenomenon of vision, without attempting to establish a method for pictorial application. The works nearest Leonardo's time were Cennino Cennini's *Il Libro dell'Arte* (1400), L. Ghiberti's *Commentarii* (1448-1455), Filarete's *De Architettura* (1461-1464), Francesco di Giorgio's *Trattato di Architettura* (1482), Piero della Francesca's *De Prospectiva Pingendi* (1485), and Leon Battista Alberti's *Della Pittura* (1435). It is uncertain whether Leonardo was familiar with the latter work, since Domenichi published it in Venice in 1547, and in Florence by Bartoli in 1548, after Leonardo's death. However, Luca Pacioli met Alberti in the latter's later years, and he met Leonardo at the time he painted the *Last Supper* [5]. It is therefore presumable that Leonardo knew of Alberti's treatise through Pacioli, and that he had knowledge of Piero della Francesca's treatise also through him, since Pacioli had been

Francesca's friend and disciple. Some authors even contend that Leonardo abandoned the idea of formalizing or finishing his treatise on perspective upon hearing of Piero's work.

In a sense, Leonardo's *costruzione legittima* is essentially the same as Alberti's [6]. In comparing Alberti's interpretations of perspective according to Grayson, Panofsky, Klein, Kitao and Parronchi [7], with Leonardo's schemes MS.A, fol. 36b; MS.A, fol. 37a; and MS.A, fol. 41a, it is found that the same principle for systematizing the representation of space is used. The difference between the two methods lies in the determination of the observer's distance point, on the perspective plane, that implicitly they contain. My geometric interpretation of the Albertian model could enhance this issue [8].

## 4 ON THE PERSPECTIVE OUTLINE

It is likely that the first compositional decision Leonardo made was to define *in situ* the position of the horizon line (5.65 m above the original floor level), so as to ensure a dominion over the outline of the mural. According to Vasari [9], the *al fresco* technique called for an initial sketch from which a more detailed drawing was made. The next step was to employ a reticulated base to prepare the cartoons on which the original drawing was enlarged and transferred in sections. The silhouettes were delineated until an integral composite scheme was obtained. The final step was to print the cartoons on the layer of plaster using an iron stylus.

Strictly speaking, Leonardo's mural is not a fresco; he used a technique of his own invention similar to oil painting applied to a damp wall [10]. However, his procedure for delineating figures could have been similar to the *al fresco* technique. This was a more convenient process because it allowed Leonardo to delineate the apostles as he obtained each from the natural version. For example, Leonardo spent so much time on Judas' face that he made the prior despair [11] and, according to Vasari, the face of Jesus is unfinished. On the other hand, it is possible that Leonardo may have at least drawn the architectural elements of the composition directly on the wall. Incisions, which helped guide the original outline of the coffered ceiling, have been discovered during the current restoration of the *Vinciano* fresco, which is being carried out by Pinin Brambilla Barcilon [12]. As shall be mentioned further on, this is physical evidence on which to base the hypothesis of the perspective outline of the cenacle. A double circular incision used to delimit the outline of the wreaths has also been discovered in the three lunettes [13]. It is also possible that Leonardo built a clay model before composing on the cartoons [14]; this technique made it easier to visualize the light and shadows cast on the figures and it improved the appreciation of the entire composite. The acceptance of any of these three alternatives: cartoons, direct application on the wall, or a clay model, leads to questions of how far from the observer the fresco was situated, and whether Leonardo first attacked the composition of the human figures or the architecture of the refectory. In any case, there is the question of how Leonardo harmonized the human figures and the architecture.

With respect to the *Last Supper*, Clark mentions that there are two studies on

composition [15], and Veltman makes reference to three [16] studies. The detailed analyses of these studies allow one to infer that Leonardo did not design the cenacle of the apostles separately. As can be seen in the first study (W12541r, 1493-1494), behind the composition of the figures is sketched some arches over columns. This suggests that even though Leonardo initially conceived the group of apostles, he had also contemplated the idea of an architectural enclosure, that is, from the outset; these were the principle compositional elements. The study of the Venetian Academy (recomposition by the author of a sketch for the *Last Supper*, 2005) [17] reveals Leonardo's concern with grouping the apostles; this is the principle problem of the pictorial composition. In works previous to Leonardo's, this problem was addressed by arranging the apostles on both sides of the table, or by endeavoring to separate Judas from the group, such as in Andrea del Castagno's *Last Supper* (a fresco in St. Apollonia, Florence, around 1450). Leonardo gave a unique solution to the problem of composition by arranging the apostles on one side of the table including the ends. In so doing, he generated a frontal symmetry with respect to the observer which is emphasized by the intense dialogue between the apostles who, following the dramatic "*unus vestrum...*", sit in clusters of three. Leonardo, in his *Trattato*, gives several advises to compose figures in historical paintings, such as: "Represent your figures in such action as may be fitted to express what purpose is the mind of each; otherwise your art will not be admirable." [18]

In the (W12541 n.d.) study, the problem of the final arrangement and attitude of the human figures is practically solved. However, even though this study pays greater attention to the architecture of the refectory, it is still notably lacking in depth, in fact, the table is not part of the first plane, and the backs of the apostles are practically propped against the rear wall. Unfortunately for my study, there is no drawing to indicate how Leonardo arrived at the final solution for the architecture of the refectory or why he radically changed his original conception [19]. My hypothesis is that Leonardo felt the need to give the imaginary cenacle greater depth. When he climbed the scaffolding to look at his painting *in situ*, and upon feeling the spatial aspect of the real refectory, Leonardo realized that the painting needed more depth. He therefore decided to work on the perspective and consequently changed the architectural design concept outlined in the third sketch until he arrived at the solution now familiar to us. This idea is not opposed to the functional explanation shared by authors such as Wölfflin, Klark and Steinberg, in the sense that the final solution arrived at by Leonardo reinforces and adds harmony to the event. And, from my point of view, what is most important is that Leonardo reconsidered and adjusted the design of the fresco (including the lunettes and the decoration of the vault) so that it belonged exclusively to its site [20], in the way that the design of *Fallingwater* (Frank Lloyd Wright, 1936) is inseparable from its site in Bear Run, Pennsylvania.

It is unlikely that Leonardo set out to model the scene from a preestablished architectural plan, as would be done today. Vasari clearly described this procedure is about 1550 [21], without determining the origin of the technique. On the other hand, the first mention of the use of the vantage point is found in Viator's *De Artificiali Perspectiva* (1505) [22]. Therefore,

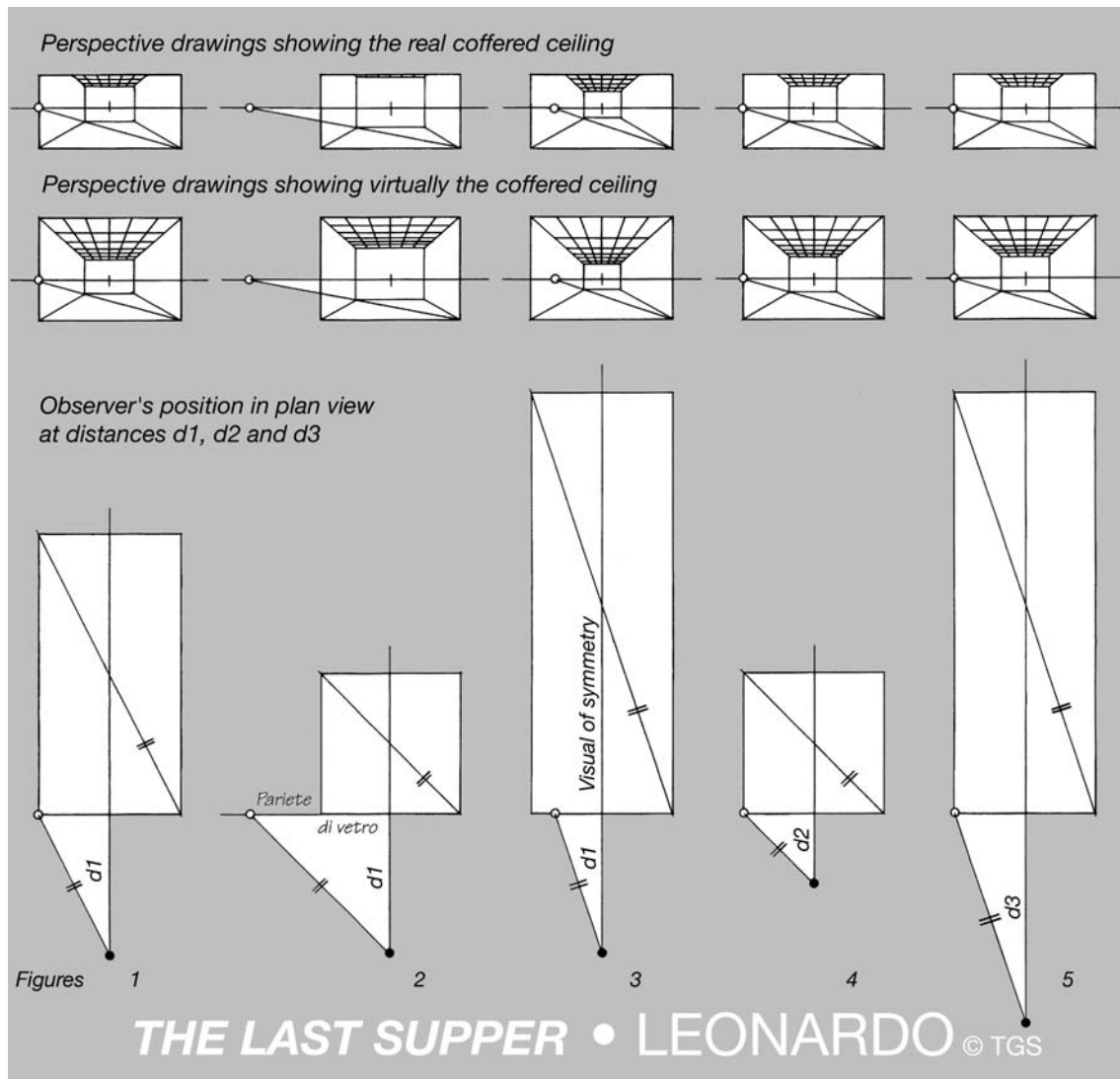
the most probable hypothesis is that the outline was applied in perspective directly to the wall, or in a sketch drawn to scale, such as Leonardo's studio for the *Adoration of the Magi* (*Gabinetto dei disegni e delle stampe degli Uffizi*, 16.3 x 29 cm, 1481), from which it is inferred that it was not taken from an architectural plan, but rather drawn directly in perspective based on the reticulation of the floor. Leonardo had previously employed reticulation as an aid in drawing a landscape of the Arno Valley (*Gabinetto dei disegni e delle stampe degli Uffizi*, 19.6 x 28 cm, 1473). He used the reticulation to orientate space and draw fields in the depth planes.

Based on the material analyzed up to this point, there are no clues as to the distance of the observer. Leonardo could not have omitted this factor since it is a characteristic element of his *costruzione legittima* (MS.A fol.41r), however, he might have fixed the observation point at a distance equivalent to half the depth of the imaginary refectory. This would have allowed him to fix the vanishing point of the ceiling diagonal on the right or left lateral border of the wall with the intent of controlling or verifying the outline *in situ*, that is, in order to control the execution directly on the wall or to verify the magnification of the work if he started from a sketch. It is impossible to fix this vanishing point beyond the border since the real refectory wall would interfere and, even if it were possible to do so [23], what advantage would there be in fixing it beyond the border or what disadvantage would there be in fixing it within the border? Both of these questions are addressed later.

Other artists influenced Leonardo and his contemporaries, and such influence is often revealed only in certain elements or details. For example, it is likely that Leonardo assimilated the effect of spatial continuity from Domenico Ghirlandaio's *Last Supper* (cenacolo di Ognissanti, Firenze, 1480), which is achieved by means of vaults painted on the upper part of the fresco, creating the optical illusion of a deeper row of arches. This illusion is emphasized by including a window in each lateral wall, whose 'illumination' is used to give the effects of light and shade to the scene; even the real console in the center of the mural is included. From this point of view, the architectural integration is better managed in Ghirlandaio's *Last Supper* than in Leonardo's. The conception of the cenacle seen frontally, placing the table transversely and seeking a rational principle in its architecture, is found in Andrea del Castagno's *Last Supper* (Cenacolo di Sant' Apollonia, Firenze, 1445), however, the spatial integration of the fresco with the real cenacle is primitive, because the fresco does not cover the entire wall and it combines a solution for the interior with the exterior. In order to handle the light and shades of the robes of the apostles, Andrea included two windows to the right of the fresco. The modulation of "six" which Andrea gives the marble panels of the rear wall coincides with the modulation Leonardo adopted for the coffered ceiling in the orthogonal direction [24]. I am not attempting to infer that these works directly influenced Leonardo, but merely pointing out that some elements in his work had already been used by other artists for similar purposes. However, there are no antecedents with respect to the compositional sense of positioning Jesus and his apostles on one side of the table. This is the truly original element of Leonardo's work.

## 5 CONSTANT DISTANCE

Before offering a hypothesis on the outline of the *Last Supper*, I shall discuss the principle issue on which this study is based, that is, the *distance* from which the mural was painted. Leonardo could have placed himself at any distance, as long as the vanishing points of the diagonals of the floor and ceiling, whatever the “real” dimensions of the imaginary refectory might have been, fell naturally on the horizon line and exactly on the limit or border of the painting, as shown in figures (1, 4 and 5). This statement is valid in purely geometric terms, but pictorially unacceptable, because if the scale of any of these three figures is adopted, the other two figures are incongruent because the scale of all the elements in the scene changes.



*Hypothesis of the simplified outline of Leonardo's illusory refectory (Cenacolo di S. Maria delle Grazie, Milano). Note that in Figures 1, 4 and 5, the same perspective result is obtained even when the plans have the different perspectives.*

For example, if the table from a given solution is transferred to either of the other two figures, it loses its normal dimensions and becomes either too narrow or too wide. For this reason, Leonardo had to determine a certain *distance* in order to make all the elements in the scene coherent.

Figures (1, 2 and 3) show the plans and perspectives of three hypotheses for the simplified outline of Leonardo's illusory refectory. If in figure (1), a line parallel to the diagonal of the refectory floor is drawn from the observer to the picture plane, the vanishing point of the diagonal will be obtained exactly at the lateral border of the mural. However, if the depth of the imaginary refectory had been less, as shown in figure (2), the vanishing point would be situated beyond this border, and therefore Leonardo would have had to devise some contrivance with which to directly apply the outline to the wall, as suggested in note [23]. If the depth of the imaginary refectory had been greater, as shown in figure (3), the vanishing point would fall within the scene. As may be observed in figures (2) and (3), there are some pitfalls in the structure of the work's composition. For example, in figure (2), the sense of depth is too shallow; it approaches the effect Leonardo wanted to avoid in sketch (W12541). In addition, the presence of lunettes in the real refectory forced the depth of the mural to a certain extent in order to give it relief on the rear wall. On the other hand, in figure (3), the sense of depth is too deep. This produces a condition of unbalance between the dimensions of the rear wall and the two lateral walls (from which the tapestries hang); the proportional balance by thirds is lost and a quarter or less of the painting is left for the rear wall. The windows are also downsized and the entry of light is reduced; this takes away from the *chiaroscuro* of the interior atmosphere of the scene, an aspect Leonardo was very concerned about.

It is important to note that in figures (1, 2 and 3), the observer is the same distance from the perspective plane. The only element that varies is the depth of the refectory, that is, strictly speaking, the onlooker is observing three different refectories with three different perspectives from the same distance.

## 6 VARIABLE *distance*

Again considering figure (1), and varying the *distance* of the observer from the perspective plane in figures (4 and 5), such that the vanishing point of the diagonal remains fixed at the border of the mural, the same perspective is obtained in all three figures (1, 4 and 5). In other words, if for any increase or decrease in the depth of the refectory (or in the *distance* between the observer and the perspective plane) the parallelism between the diagonal of the refectory and the line running from the eye of the observer to the vanishing point of such diagonal is preserved, the perspectives will always be identical. This principle of parallelism in perspective corresponds to the simplest case of *anamorphic* projection for parallelogramic bodies [25], which occurs when the symmetry line of sight is perpendicular to two of the six sides of such body.



For the case analyzed in figures (1, 4 and 5), it may be concluded that regardless of the visual distance adopted, the depth of the imaginary cenacle will always be twice such *distance*.

As previously mentioned, the first statement of the problem was a matter of observing three different refectories from the same *distance* and this resulted in different perspectives. In the second statement of the problem, there were also three different refectories, but variations in the distances resulted in exactly the same perspectives. However, in the first statement, it is possible to include the case in which three different refectories observed from the same *distance* result in the same perspective. This can be achieved by employing the principles of accelerated perspective such as the illusionary room for coupling images, or the perspective room invented by Phillipe Comar [26], in which two halves of a scene with different depths are perfectly coupled so as to appear as one when observed from the same vantage point.

If the principle of anamorphic perspective is followed, there is no sense in wondering what the true depth of the refectory would be, since there are dozens of solutions. This dilemma makes it inevitable to resort to Leonardo's practical sense, and assume that the *distance* he chose was not greater than the dimension afforded by the scaffold floor, since it would not be very practical to build a large scaffold in order to satisfy a big observation *distance* (perhaps once or twice the width of the mural). A large scaffold would have hindered the use of the refectory for over two years; Matteo Bandello would surely have mentioned such an inconvenience in *Le Novelle*.

## 7 NATURAL DISTANCE VS. FOCAL DISTANCE

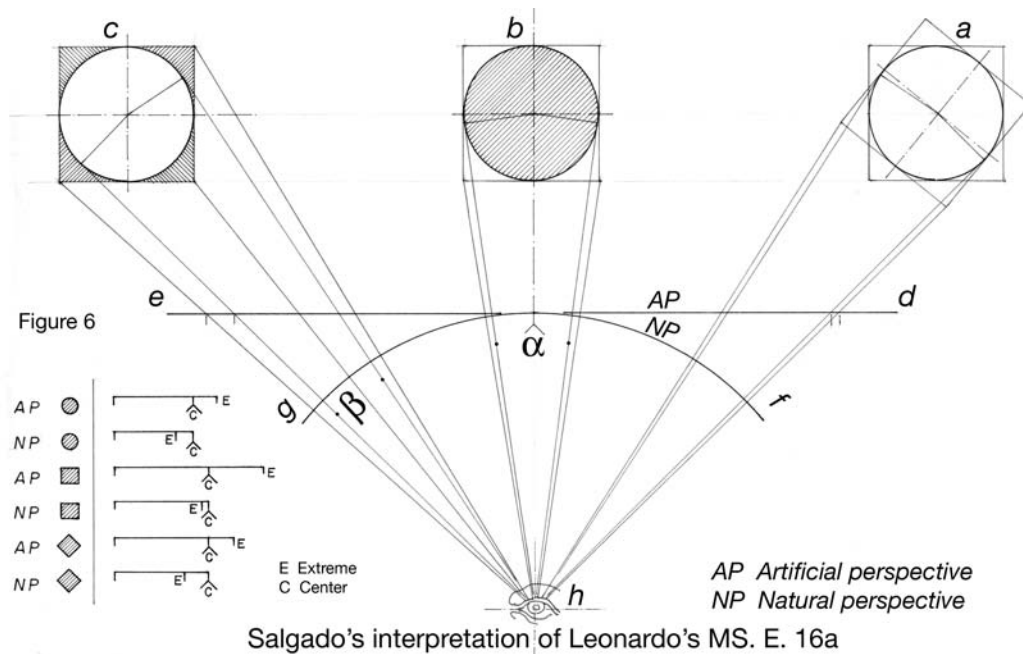
In Albertian perspective, the observer *distance* is determined by the interval between the eye and the picture plane, and the image formation in Modular Perspective [27], as the *focal distance* [28], is defined by the aperture of visual field. The concept of *focal distance* was not explored in Renaissance perspective, thus making it impossible to unequivocally answer the question put forth at the outset of this work with respect to the *distance* Leonardo chose to carry out the perspective outline of the *Last Supper*, and therefore it is also impossible to determine the "real depth" he gave the imaginary refectory. As shown in figures (1, 4 and 5), the rear of the imaginary refectory depends solely of the *natural distance* of observation and not on the aperture of visual field, that is, it does not depend on the *focal distance* of image formation.

It is important to distinguish between the aperture of visual field and the visual angle of observation. The first concept refers to the visual capability of the human eye, while the second concept refers to foreshortening. Leonardo stated a challenging problem that he illustrated with a small sketch representing three round columns (MS. E, fol.16a). In this scheme, Leonardo compared his projections in artificial and natural perspective. Certainly, he sought to understand the geometric nature of marginal distortions since, from his sketch, it is evident that in artificial perspective the outer columns become larger when projected on



the plane surface, and that in natural perspective they become smaller when projected on the curved surface [29]. If the columns in the original scheme are made square and compared to each other (see Figure 6), one concludes that this is a specific case of foreshortening, since the shape and orientation of the bodies was not taken into account. Note that in Fig. 6 the front sides of the three square columns project in artificial perspective the same diminishing on the plane surface. The perspective of a cylindrical column with a curved and continuous surface is different from the perspective of a square column with a polyhedral surface.

With respect to the nature of marginal distortions, when the *Last Supper* is observed from a far distance, there is a noticeable widening of the bodies of the apostles situated at the ends of the table, and when the mural is observed from a *distance* equivalent to half its width, such anatomical deformations disappear. This visual evidence is congruent with Leonardo's own observation that in order for a work to be convincing when viewed from a short distance, it is necessary that the observer stand exactly at the vantage point from which it was painted and, to the contrary, if the painting is to be observed by many people at the same time, it must be painted from a distance at least ten times its width. This leads to defining the visual field angle as an angle controlled by the eye's focal distance, that is, by the human focal distance. Herein lies the importance of introducing and studying this concept in current perspective theory.



*Reinterpretation of Leonardo's drawing MS.E. fol. 16a, in which the apparent size of bodies is analyzed in relation to the visual angle ( $\alpha$  vs.  $\beta$ ), the type of projection (natural vs. artificial), and the shape of the body (circular vs. square).*

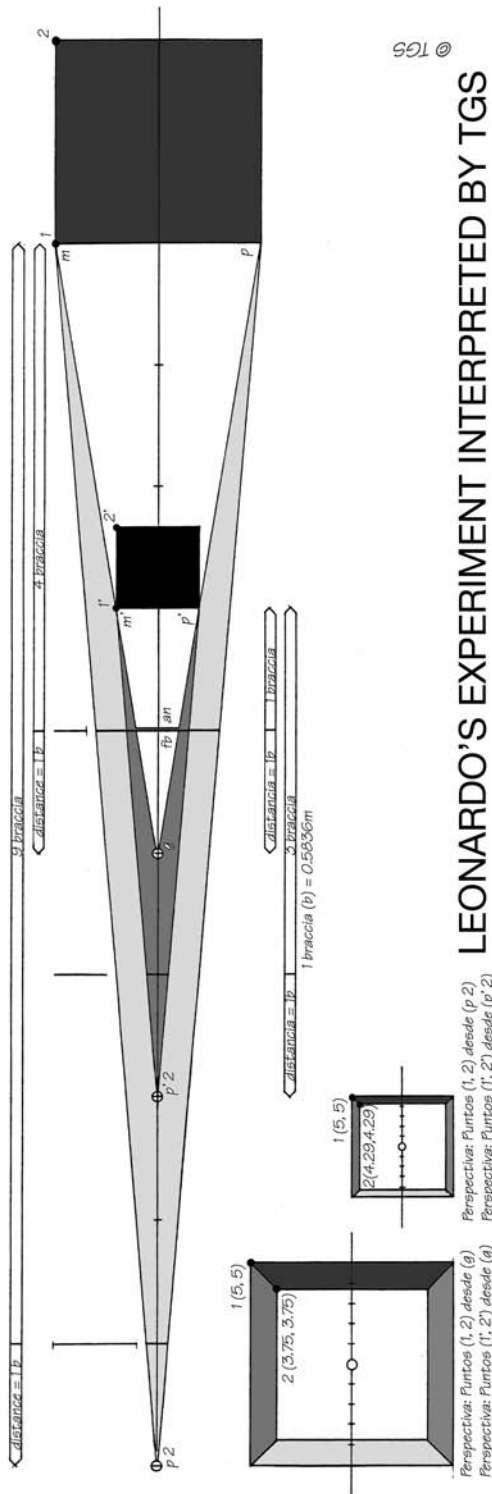


Figure 7. In the frontal perspectives of points (1, 2), (1', 2') from (G), and of the same points from (P2), it is possible to appreciate the variation in the formation

of the image of the two cubes, that is, not only does the apparent size of the cube change, so does its perspective construction.

## 8 LEONARDO'S EXPERIMENT

Based on a scheme by Leonardo, in which he studied the relationship between the size of an object and the distance of the observer (Codex Atlanticus 42 rc), Kenneth Keele makes a zealous reproduction of the experiment implicit in such scheme [30], confirming Leonardo's thesis on visual pyramids, in which the height of the object represented by its base is inversely proportional to its height, that is, to the distance between the observer and the object. The importance of the experiment lies in its very purpose: to measure the apparent diminishing in size of objects. However, an equally important factor not considered by Leonardo and absent from Keele's interpretation is the variation in *image formation* in this experiment.

This interpretation of Leonardo's experiment leads to the conclusion that the distance of the observer not only affects the apparent size of objects but also their image formation. In figure (7), a comparison of the perspectives resulting from the same experiment reveals that, despite being the same size, the objects have different image formations. This is due to variations in the distance between the observer and the object in each case. Applying the Modular Perspective Method to figure (7), and establishing that in both cases the projective plane of the visual field measures  $10m$ , that is,  $(+5 m)$  from the origin to the right and  $(-5 m)$  from the origin to the left, the perspective coordinates of points 1 and 2 of the body (mp) as seen from (g) are: 1 (5.00, 5.00), 2

(3.75, 3.75), and the perspective coordinates of the same points of the body (mp) as seen from (p2) are: 1 (5.00, 5.00), 2 (4.29, 4.29). This demonstrates that the perspective of the body (mp) as seen from (g) has different image formation as compared to the perspective of the same body as seen from (p2). As a corollary to Leonardo's experiment, it is possible to establish that the bodies (mp) and (m'p') could have the same image formation, in the perspective plane, when projected from the same vantage point, since they are proportional and there exists a correspondence between each of their parts.

The problem behind the outline of the *Last Supper* was not the calculation of the reduction in size, because the refectory wall itself defined the size of the first plane of the scene. The problem lay in predetermining the most suitable image formation which would avoid distortions of the first planes while at the same time recreating an enclosure of real dimensions. Folio CA 42 rc (1478-1490) preceded the *Last Supper* so it may have influenced the proportions of the outline; coincidentally the rear wall of the refectory is exactly one third the width of the pictorial frame which is the same proportion of the object observed in the experiment.

The previous experiment may be inverted by taking the image already projected on the picture plane and, from a given observation point there, endeavoring to reconstruct the dimensions of the refectory. If for a moment, one accepts the idea that the fresco should be observed from the Prior's table placed opposite the mural, one must respect the proportions of the perspective outline given in the picture plane, in which both the lateral walls and the rear wall each occupy one third of said plane in the horizontal direction. Given that these proportions are fixed in the picture plane, if they are observed from the Prior's table by means of lines of sight which form the interior envelope of the imaginary refectory, that is, until they reach the rear wall, the depth would be approximately 70 m, that is, twice as long as the real refectory. This is architecturally unacceptable; the plan would be longer than the one shown in figure (5).

Leonardo analyzed in detail the behavior of size reduction using the lines of sight pyramid principle. For the *Last Supper*, Leonardo knew that if he applied a 1/3 reduction to the rear wall in the picture plane as of "the central line of vision" [31], the depth of the wall was exactly twice the distance that lay between the observer and the picture plane. In terms of the fresco's real dimensions, this situation is as follows: The central line of vision is situated 4.425 m above the horizon line, 4.425 m is half the total width of the fresco; 1.475 m from this line (1.475 m is one third of 4.425 m), be it on the right or on the left, is the outline which indicates the reduction in size of the rear wall, so that in order to find the depth of the wall the line of sight running through the line at 1.475 m is produced from the observer until it reaches a distance equivalent to twice the distance between the observer and the picture plane. This is proven mathematically as follows: let

$$a = 1.475 \text{ m}$$

$d$  = the distance between the observer and the picture plane

$\alpha$  = the angle formed by any visual with respect to the central line of vision



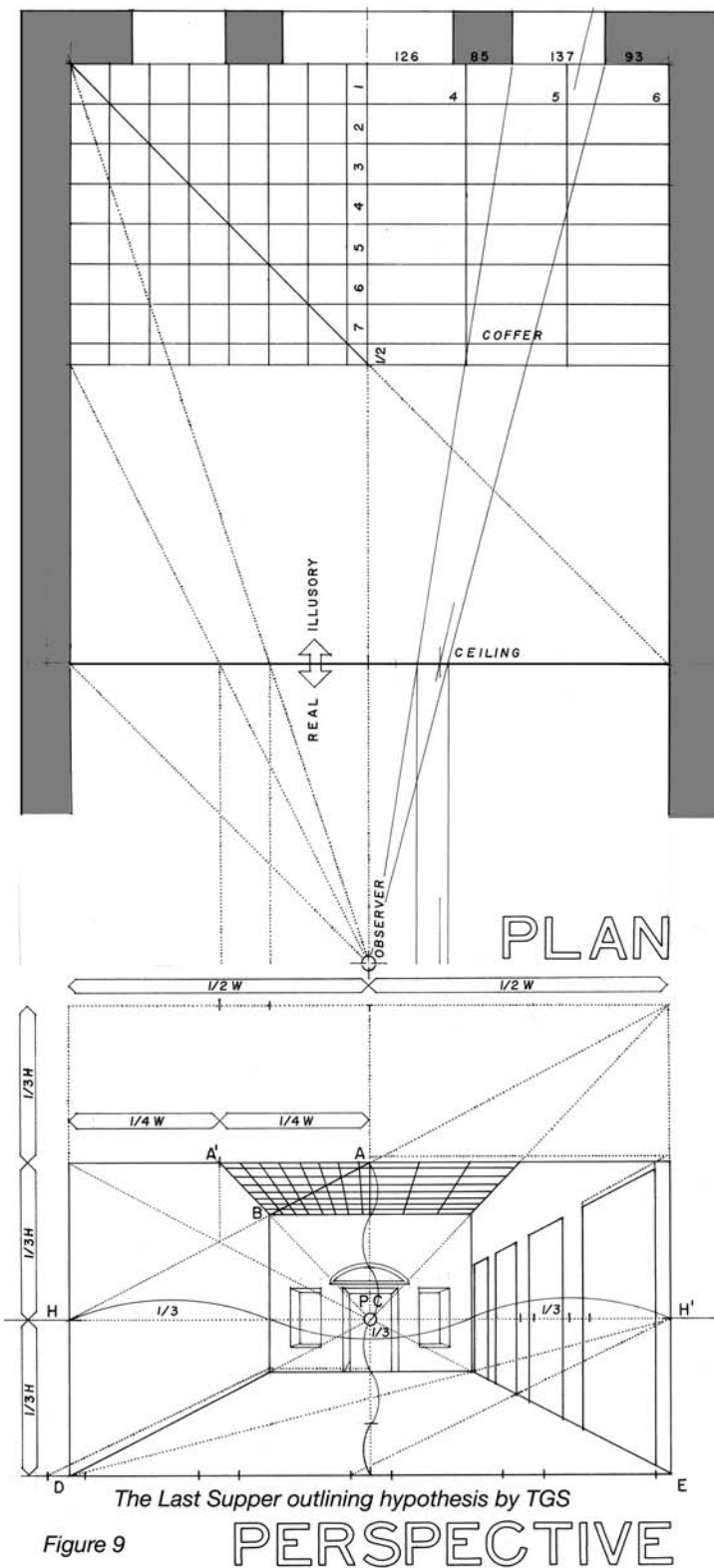


Figure 9

across the mural. The point (A) determines the midpoint of the upper envelope of the mural.

4. In this manner, Leonardo divided the mural into four parts. Since Leonardo was left-handed, for convenience, it is likely that he chose the upper left-hand quadrant to draw the ceiling, following a variation on the *costruzione legittima* [34]. If this hypothesis is true, it is obvious that there was sufficient room in which to draw the ceiling from the vantage point and therefore Leonardo could have employed the diagonal (A-B) to draw his outline, “*a-b e lla ripruova*” (MS. A, fol.41r).

In addition to the Albertian idea of pavement modulation, Leonardo demonstrated the use of the *costruzione legittima* [35] to foreshorten a ceiling plane (MS. A. fol. 37a, 1492). This correlation allows one to assume that the entire coffered ceiling was conceived as a large picture that was later subdivided into four parts in order to obtain the final outline of the coffered ceiling. It is unlikely that Leonardo drew the entire coffered ceiling (as suggested by some authors), because he would have had to fix at least two points at a height 5.63m above ‘*il ponte*’. This would have been undesirable because the points would have

fallen on the curved surface of the lunettes where it would have been difficult to control the accuracy of the drawing.

5. Leonardo fixed the point of the diagonal (A-B) at (H), the utmost left point on the horizon line; and in order to determine the height of the rear wall of the imaginary refectory (which is also the corner of the ceiling), Leonardo again divided in two this upper left-hand quadrant by using its diagonals, one of which is the line (H-A). This is how Leonardo obtained the point (A'), which he then vanished at the central point (PC), where the point (B) is the intersection of the lines (H-A) and (A'-PC).

6. The points (H) and (H') define the vanishing points of the diagonals of the ceiling, that is to say, the lines (H-PC) and (H'-PC) determine the vantage point equal to 4.425 m.

7. The modulation of the coffered ceiling is 6 caissons in the front direction, by 7.5 visible caissons in the direction of depth (15 caissons if complemented with the virtual outline of the coffered ceiling). It is difficult to know which process Leonardo used to modulate the coffered ceiling. However, I tend to assume that he exercised his wise and ample judgment as a painter to create an effect of 'sufficient depth' in the little space available, that is; he limited the number of visible beams. For depth, Leonardo chose the odd number (15), because the fact that the mural's picture plane intersects the ceiling, on the line of empty caissons and not on the beam, makes evident that he tried to avoid binding the architrave with the beam. In order to draw the 7.5 depth modules, it is sufficient to divide the line (A-A') into 7.5 equidistant modules and produce their vanishing lines to the point (PC); the corresponding depth of each module is obtained by means of the intersection of each vanishing line with the diagonal (A-B).

8. The floor is demarcated by drawing the vanishing lines (PC-E) and (PC-D). If the tapestries and their sequence are drawn on the line (D-E), whose magnitude is real because it is in the first plane, it is sufficient to refer these dimensions to point (H'), and where they intersect the diagonal (PC-E) will determine the depths along the right wall. Likewise, referral to the point H will determine the depths along the right wall. Alternatively, once the depths along either wall have been obtained, the depths along the opposite wall may be obtained by drawing parallel lines. In this case, in contrast with the ceiling, the floor is complete (for purposes of the outline) and, for this reason, the diagonal (D-F) runs from one corner to the other, spanning the entire width.

## Visual Experiment

In concluding this presentation, and in order to provide simple illustrations of *natural distance* and *focal distance*, I invite the reader to follow the description of the experiment described below. Based on a drawing of the refectory made expressly for this purpose (Figure 10), in which appears the entire coffered ceiling, and where the human figures are eliminated and the other elements of the scene are merely outlined so that the entire

geometry of the cenacle may be appreciated, the following experimental hypothesis is put forward: a) If figures (1, 4 and 5) correspond to the perspective of the drawing, then each must be observed from the distance for which it was designed. b) The observations must be made by accommodating the principle line of sight on the central vanishing point, while ensuring that the drawing is perpendicular to the line of sight.

It is recommended that each observation be made with only one eye [36], and, if possible, the drawing should be masked with black cardboard, in order to isolate it and create a feeling of “being part of the scene”. c) Considering the size to which the drawing is to be printed, and with the purpose of reproducing to scale the observation conditions, the following consideration is made: If the printing width is to be 13.6 cm, then figure (1) should be observed from a distance of 13.6 cm, figure (4) should be observed from 6.8 cm, and figure (5) should be observed from 20.4 cm. d). The perception of the drawing from the three distances mentioned, will lead the reader to form his or her own conclusions.

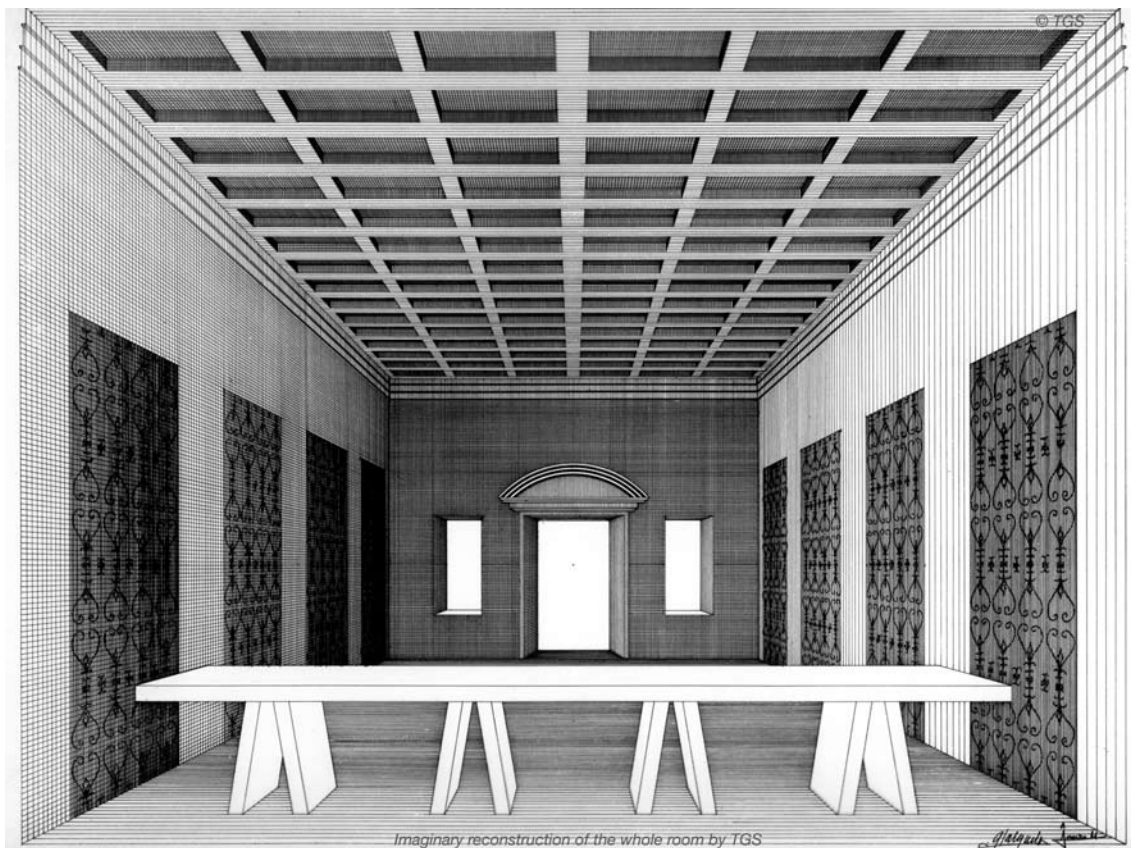


Figure 10

*Ideal representation of the refectory without the lunettes. Drawing by TGS and JME*



However, my comments on the experiment are the following: In figure (1), a sense of equilibrium is perceived, since the depth of the refectory is approximately a one (front) to two (rear) ratio; on the other hand, in figure (4), the perception of the refectory space is that of a cube (the front, the rear and the height maintain a one to one ratio), which is emphasized by the effect of the entire coffered ceiling. The author is convinced that this is the correct observation *distance*.

After these two observations, it is advisable to rest one's eyesight before observing figure (5). The perception in figure (5) is really one of greater depth than figure (1); this effect can be increased by covering half the ceiling with a piece of black cardboard to create Leonardo's original version. The nature of this experiment goes beyond the principles of the *costruzione legittima* and leads to a new question: Which of these three perceptions is most compatible with human vision? The concepts of *focal distance* and perceived image formation are useful in addressing such a question.

## 10 BIBLIOGRAPHY AND NOTES

[1] My research on perspective is presented in two works: *Perspectiva Modular Aplicada al Diseño Arquitectónico* (Modular Perspective Applied to Architectural Design), Vol.2, (Mexico: UNAM Press, 1982), which discusses the theory behind the method. On page 18 appears my version of the *costruzione legittima*. The second work is the article: "A Modular Network Perspective Model vs. Vectorial Models", *Leonardo*, 21, 3, pp. 277-284, 1988, which is about the perspective model of my method.

[2] Leonardo left several outlines of his *costruzione legittima* indicating the *velo* for measuring distance and the diagonal for verifying the coherence of the *pavimenti*, but in practice he may well have abbreviated the procedure by obtaining the vanishing point of the diagonal. In my experience as a theoretician of perspective, I can say that when I explain my method, I do so step by step without abbreviation. However, when I apply my method, I adapt it to the needs of the outline and sometimes this adaptation may seem to differ from the theory.

[3] John White, *The Birth and Rebirth of Pictorial Space* (London: Faber and Faber, 1957), p. 209: "... [Leonardo] was thoroughly excited by his discovery and intended to publish it in a separate book. Unfortunately Leonardo's manuscript was stolen before he could do so. This was doubly a misfortune, as the original notes for it have also disappeared."

[4] Francis M. Naumann, "The *Costruzione Legittima* in the Reconstruction of Leonardo Da Vinci's Last Supper", *Arte Lombarda*, Milano, n.s., vol. 52, 1979, pp. 63-89. On pages 66-69, Naumann describes the treatises Leonardo was acquainted with.

[5] Kenneth Clark, *Leonardo da Vinci* (Great Britain: Penguin Books, 1993), p. 157: "He [Pacioli] arrived in Milan in 1496, and we know from Leonardo's notebooks that the two men were soon on intimate terms. By 1497 they were collaborating on the *Divina Proportione*."

[6] Kenneth D. Keele, *Leonardo Da Vinci's Elements of the Science of Man* (New York: Academic Press, 1983), p. 48: "Eventually he reproduces the geometrical figure of perspective described but not drawn by Alberti. Thus, Leonardo comes to make the first drawings of Alberti's perspective with the vanishing point on the horizon at the level of the eye. He has constructed all the elements of Alberti's '*costruzione legittima*' from his own observations step by step, himself."

[7] Franco Borsi, *Leon Battista Alberti, The Complete Works* (Italy: Electa Milano, 1986), p. 199.

[8] T. García-Salgado, “*Geometric Interpretation of the Albertian Model*”, Leonardo [USA: MIT Press] Vol. 31 No 2, pp. 123-128, 1998.

[9] Giorgio Vasari, *Vasari on Technique* (New York: Dover Publications, Inc. 1960), p. 215: “Afterwards, for transferring the outlines on to the said piece [cartoon], the artist proceeds to impress them with an iron stylus upon the coat of plaster, which, being fresh, yields to the paper and thus remains marked.”

[10] Clark [5] p. 146: “Such irregular methods meant that the painting could not be *al fresco*; and in fact, we know that Leonardo used a medium containing oil and varnish. The wall was damp and as a result the painting very soon began to suffer.”

[11] Giorgio Vasari, *Lives of the Most Eminent Painters, Sculptors & Architects* (London, W.: Philip Lee Warner, Publisher to the Medici Society, Limited, 1912-14), Vol. IV, pp. 96-97: “It is said that the Prior of that place kept pressing Leonardo, in a most importunate manner, to finish the work... However, he would seek out a model for the latter; but if in the end he could not find a better, he should not want that of the importunate and tactless Prior.”

[12] An interview the author had with Dottorosa Pinin Brambilla Barcilon on February 19, 1994, at the S. Maria delle Grazie refectory. For further documentation on the restoration of the Leonardo cenacle, refer to: P. Brambilla Barcilon, *Il Cenacolo di Leonardo in Santa Maria delle Grazie. Storia, condizioni, problemi*, “Quaderni del restauro”, 2, Milano, 1984.

[13] Pinin Brambilla Barcilon and Pietro C. Marani, *Le Lunette di Leonardo nel Refettorio delle Grazie*, Quaderni del Restauro 7 (Milano: Olivetti, 1990), p. 53

[14] Vasari [9] p. 214: “Many masters also before making the composition of the cartoon, adopt the plan of fashioning a model in clay to see the projections, that is, the shadows caused by a light being thrown on the figures...”

[15] Clark [5] p. 150: “The steps by which Leonardo arrived at his final solution are lost to us. We have very few drawings for the Last Supper, and for the composition only two studies. The more elaborate of these, a red chalk drawing in the Venice Academy, is one of the most puzzling of all Leonardesque relics.”

[16] Kim Veltman, *Studies on Leonardo Da Vinci I, Linear Perspective and the Visual Dimensions of Science and Art* (München: Deutscher Kuntsverlang, 1986), p.339: “Leonardo’s earliest study for the *Last Supper* on W12542r, (pl.6.1, c.1495) is almost entirely without spatial hints. A second study (pl.6.2, c.1495) indicates foreshortening in the table and bears comparison, as Goldscheider has pointed out, with both Andrea del Castagno’s fresco (Florence, S. Apollonia) and Francesco Botticini’s predella panel (Empoli, Galleria della Collegiata) devoted to the same theme. A third sketch on W12541 (pl. 6.3, n.d.) is much more developed as regards spatial effects.”

[17] T. García-Salgado, “*Form in Site and Perspective*” in *Aesthetics & Architectural Composition Proceedings of the Dresden International Symposium of Architecture 2004* (Germany: pro Literatur Verlag, 2005), pp. 61-62: “In my understanding as a draftsman, the Accademia sketch could be the key to solve the mystery. As the sketch’s paper sizes 29 x 39.2 cm, too short to contain the apostles’ table in one piece, the scene was split in two. However what really matters here is what the artist may have envisioned through the sketch. By recomposing the split-sketch into a single one, as I did it in (Fig. 4), we probably may be able to see what was on the artist’s mind... and it looks more like a unified scene; strongly resembling the final composition, as we know it today.”

[18] Jean Paul Richter, *The Notebooks of Leonardo da Vinci*, Vol. I (New York: Dover Publications, 1970), p. 300.

[19] Clark [5] p. 151: “Between these sketches and the final composition an immense labor must have intervened; but unfortunately the drawings and studies in which the great construction gradually took its inevitable shape are almost entirely lost.

[20] Leo Steinberg, “*Leonardo’s Last Supper*”, *The Art Quarterly*, Number 4, pp. 297-409 (1973). Steinberg arrives at the same conclusion via a different reasoning: “The persistent omission or alteration of Leonardo’s perspective scheme in copies and adaptations thus emerges as negative proof that the fresco, in its totality, was conceived in and for its site.” p. 349.

[21] Vasari [9] p. 214: “... for of course when the artist has drawn out the perspectives in the small designs, taking them from the plan and setting up the elevations with the right contours, and making the lines diminish and recede by means of the intersections and the vanishing point, he must reproduce them in proportion on the cartoon.”

[22] To determine the vantage point in Viator’s construction, it is sufficient to produce the diagonal (*Linea dyametralis*) of the base square until it meets the horizon line (*Linea pyramidalis*), such that the interval between this point of intersection and the observer as seen frontally determines the distance of the observer.

[23] Assuming that he drew directly on the refectory wall and that he did not base the diagonal of the ceiling on the edge of the mural, be it because he did not follow this procedure or because he used a greater distance (10.29 m, as suggested by Naumann), there is the bold possibility that he produced the outline on the side wall of the refectory given that, despite the wall forming a right angle with respect to the mural, it does not alter the outline procedure.

[24] T. García-Salgado, “*Andrea del Castagno, Ultima Cena*”, *Ciencia Ergo Sum*, Vol. 12-3, pp. 291-298, 2005. Here I described Andrea’s Last Supper outlining at Saint Apollonia Convent (1447-1450). Even though both Andrea’s and Leonardo’s frescoes have an illusory effect they have different perspective constructions, since Andrea’s was taken from a very distant vantage point.

[25] Keele [6] p. 46: “... the obliquity of the plane of glass led Leonardo to appreciate perspectival ‘distortions’ or anamorphosis”. It is difficult to determine how far Leonardo developed these ideas, and it is even more difficult to assume that they had an influence on the *Last Supper*.

[26] T. García-Salgado, *Instrumentos para la Geometría Perspectiva* (México: FA, UNAM, 2004). Here I present several photos of Comar’s illusory room at La Cité des Sciences et de L’industrie in Paris, pp. 68-71.

[27] T. García-Salgado, *Manual de Perspectiva Modular* (A Modular Network Perspective Handbook), (Mexico: Trillas, 1991). On p. 54 are examples of controlled image formation for 45°, 60°, 75° and 90° apertures of visual field (AVF).

[28] T. García-Salgado, “*The Concept of Distance in Classic Perspective and Modular Network Perspective*”, *Proceedings of CIB Congress* (International Council of Building Research), Washington, D.C., Vol. 7, 1986, p. 3008: “... in contrast to MN (Modular Network Perspective), demonstrates that the distance to locate the picture plane in natural perspective projection is not obtained by the relation  $AVF/d = M/df$ ... The concept of focal length permitted the posing of the following general cases of the formation of image for the MN model: a) telescopic, b) angular deformation, and c) proportion of the perspective plane of visual field, the last corresponding to natural projection”. The relation between the aperture of visual field (AVF) and the distance ( $d$ ) of the observer to the perspective plane is precisely the *focal distance*. Where (AVF) is the aperture of visual field, ( $d$ ) is the distance, ( $M$ ) is the absolute value of the perspective plane equal to 10 modules ( $m$ ), and ( $df$ ) is the focal distance.

[29] Richter [18], p. 63: “And let the plane  $d e$  on which are seen 3 equal circles which are beyond this plane  $d e$ , that is the circles  $a b c$ . Now you see that the eye  $h$  sees on the vertical plane the section of the images, largest of those that are farthest and smallest of the nearest.” But as we saw through Fig. 6, the front sides of the three square columns project in artificial perspective the same diminishing on the plane surface, which seems to contradict the original premise of Leonardo’s scheme. This intriguing passage of Leonardo became a riddle since he do not specified what the letters  $g f$  are referring to. The following passage seems to correspond to plane  $g f$  instead of plane  $d e$ : “By natural

perspective I mean that the plane on which this perspective is represented is a flat surface, and this plane, although it is parallel both in length and height, is forced to diminish in its remoter parts more than in its nearer ones.” (p. 63) But here a ‘flat surface’ is described instead of a ‘curved surface’.

[30] Keele [6] pp. 77-78: “If *an* goes 3 times into *fb*, *mp* will do the same into *pg*. Then go backwards so far that *cd* goes twice into *an*, and *pg* will equal *gh*. And *mp* will go into *hp* as often as *dc* into *op2*”. Leonardo describes a variant of this experiment I; he writes: “Linear perspective deals with the action of the lines of sight proving [or testing] by measurement [a *provare per misura*] how much smaller a second object is than the first ... I find by experience that if a second object is as far beyond the first as the first is from the eye, although they are of the same size, the second will seem half the size of the first” (BN 2083 23 n, A 103 r).

[31] Keele [6] p. 49: “This central line becomes of great importance in his theory of vision”.

[32] According to E. Möller’s survey (*Das Abendmahl des Lionardo*), Baden-Baden, 1952), originally there were two doors in the northern wall, a 1.60 x 2.50 m door in the center and a 1.00 x 2.00 m door to the left. Due to flooding, the refectory floor was raised 1.00 m (approx.), and in 1652 the central and bottom portions of the mural were destroyed in order to enlarge the central door to 2.00 x 3.15 m. By this time, the small door had been eliminated. This was a terrible decision made by the Prior, since a 2.00 m rise would have avoided this irreparable damage to the fresco.

[33] This is the term Matteo Bandello (*Le Novelle*, Laterza, Bari, 1910, Vol. 2), employs in referring to the scaffold Leonardo used to paint the fresco.

[34] The reasoning he might have chosen for the partition of the ceiling was based on the Ms. A. fol. 37a.

[35] Naumann [4] p. 74: (“a situation very similar to the case encountered by the visible ceiling beams in the *Last Supper*”).

[36] Robert Smith, “*Natural Versus Scientific Vision: The Foreshortened Figure in The Renaissance*”, *Gazette des Beaux Arts* (Paris, Oct. 1974), p. 242: “As it happens, the advantages of binocularism are that it enables us to perceive the solidity of objects by seeing them ‘in the round’, and that it helps us to gauge the relative distances of solid objects *in space*. Objectively, binocularism has no advantage over monocularism in viewing objects *depicted on a plane surface*...”

## 11 GLOSSARY

*Coffered Ceiling*. Artesonado (Spanish), Plafond a caisons (French), Kassettendecke (German), Soffitto intavolato (Italian).

*Image Formation*. In Modular Perspective, the focal distance (fd) defines the interval between the observer and the *perspective plane* (PPL) where the image is formed. If (fd) is shortened, then (PPL) increases and the perspective tend to the wide-angle effect. If (fd) is lengthened, then (PPL) decreases and the perspective tend to the telescope effect. The formation of a body’s image is therefore a function of the focal distance chosen.

This article must be cited as follows:

Tomás García-Salgado, “*The Perspective of Leonardo’s Last Supper*”  
<perspectivegeometry.com> *e-journal*, Number 3 (2005).